Ci 9
(289/1960)

UDC 536.2:697.133:69.025.1

ACTA POLYTECHNICA SCANDINAVICA

CIVIL ENGINEERING AND BUILDING CONSTRUCTION SERIES No. 9

OLAVI VUORELAINEN

The temperatures under houses erected immediately on the ground and the heat losses from their foundation slab

Finnish Contribution No. 17

Also published as PUBLICATION No. 55 OF THE STATE INSTITUTE FOR TECHNICAL RESEARCH, FINLAND.

HELSINKI 1960

ACTA POLYTECHNICA SCANDINAVICA

. . . a Scandinavian contribution to international engineering sciences

Published under the auspices of the Scandinavian Council for Applied Research

in *Denmark* by the Danish Academy of Technical Sciences

in *Finland* by the Finnish Academy of Technical Sciences, the Swedish Academy of Engineering Sciences in Finland, and the State Institute for Technical Research

in *Norway* by the Norwegian Academy of Technical Science and the Royal Norwegian Council for Scientific and Industrial Research

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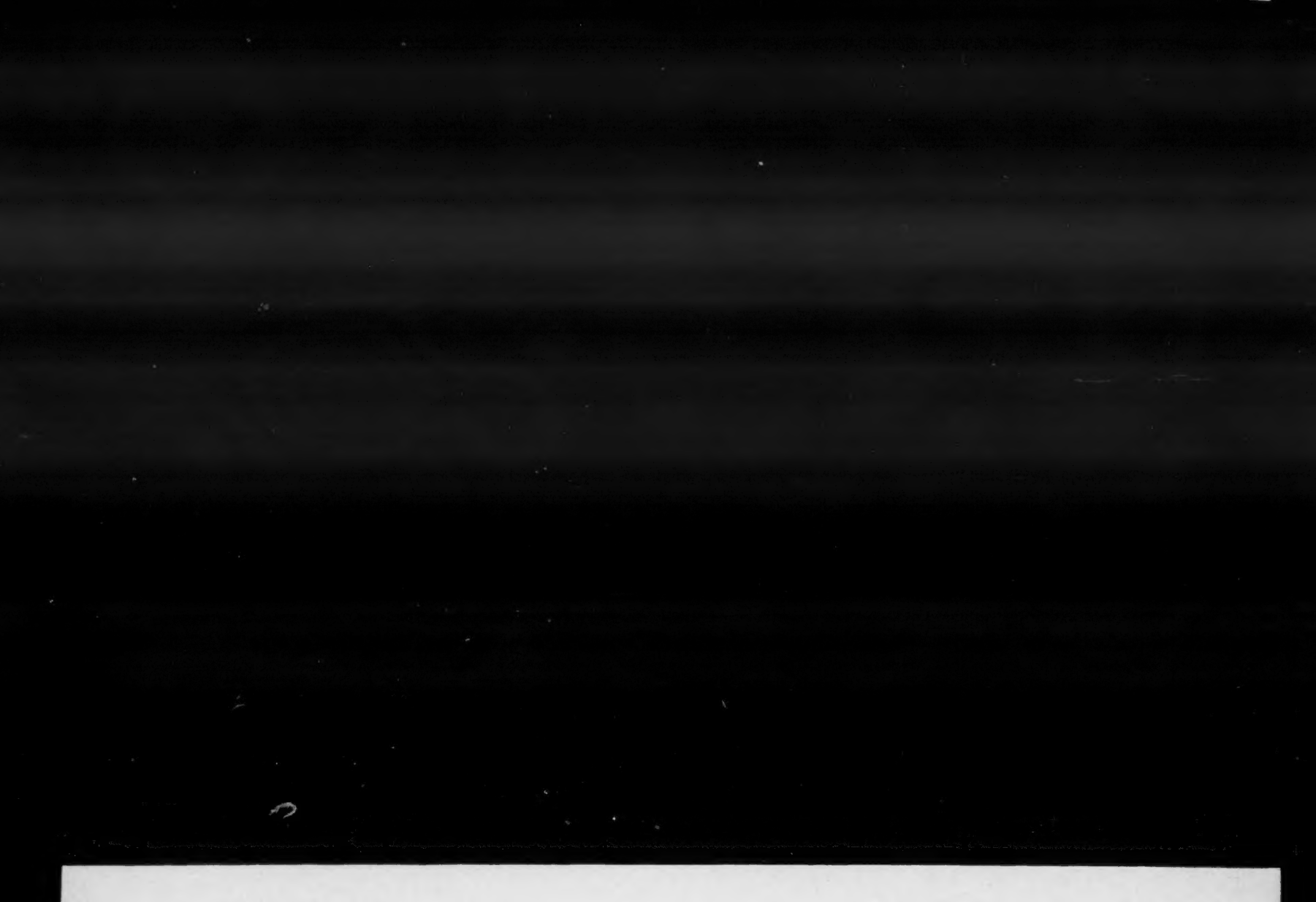
Stockholm 5 Phone 67 09 10

This issue is published by
THE STATE INSTITUTE FOR TECHNICAL RESEARCH
Helsinki, Finland



UDC 536.2:697.133:69.025.1

VALTION TEKNILLINEN TUTKIMUSLAITOS
STATENS TEKNISKA FORSKNINGSANSTALT
THE STATE INSTITUTE FOR TECHNICAL RESEARCH, FINLAND

JULKAISU 55 PUBLICATION

THE TEMPERATURES UNDER HOUSES ERECTED IMMEDIATELY ON THE GROUND AND THE HEAT LOSSES FROM THEIR FOUNDATION SLAB

OLAVI VUORELAINEN

Thesis for the degree of Doctor of Technology accepted by the Finland Institute of Technology

HELSINKI 1960

VTT Rotaprintpaino 1980

338/4

PREFACE

The first doctor's thesis in the field of heating technology to be published in Finland, this work is an account of studies carried out in the State Institute for Technical Research and enabled by a grant received from the Asuntotutkimustoimikunta (Dwelling Research Commission).

It has been possible to perform this investigation without incurring prohibitive expenses by applying a method of mathematical analysis and by using as experimental objects some of the one-family houses built at Tapiola, near Helsinki, by the Asuntosäätiö (Housing Foundation) and the building of the Secondary School at Tapiola.

At this opportunity, I wish to offer my thanks to the Director-in-Chief of the State Institute for Technical Research, Professor E. Wegelius and to the Chief of the Laboratory of Heating Technology, Professor T. Tuomola for their favourable attitude towards my work and for the inclusion of this report in the publications of the institute.

To the Asuntotutkimustoimikunta (Dwelling Research Commission) I feel obliged for the economic support which has been granted this work.

I am particularly indebted to Professor E. Laurila, by whose kind assistance this work has obtained the character of a thesis for the doctor's degree and whose valuable advice has helped me during its progress.

Mr. E. Niskanen, Ph.D., has read the manuscript and given valuable advice concerning its mathematical section, for which my gratitude is due to him.

Furthermore, I wish to acknowledge the assistance received in the gathering and treatment of the extensive observational material from several students in the Institute of Technology, in particular Mr. J. Puustinen and Mr. K. Kolu, and the contribution of Mr. O. Varho, M. Sc., who performed the required numerical calculations on the electronic computer.

Mr. U. Attila, M.Sc., has translated the manuscript with considerable skill, for which I give him my thanks.

Helsinki, May 1st, 1960.

The Author

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LIST OF SYMBOLS

A	Area, m^2
G	Thermal conductance, kcal/h ^oC
K	Conversion ratio
L	The circumference of the building, m
Q	Thermal flow; Heat losses, kcal/h
Q_o	Upward thermal flow from the ground in the region covered by the slab, kcal/h
$Q_1(t)$	Downward heat flow from the heating tube level, as a function of time, kcal/h
$Q_2(t)$	Upward heat flow from the heating tube level, as a function of time, kcal/h
Q_C	Heat losses by conduction from the house, excluding its floor slab structure, kcal/h
Q_E	Heat losses from the edges of the slab directly into ambient air, kcal/h
Q_G	Heat losses into ground from the insulated slab, kcal/h
Q_{G_o}	Heat losses into ground from the uninsulated slab, kcal/h
Q_V	Heat losses by air leakage and ventilation, kcal/h
Q_c	Heat introduced in the concrete slab, measured by calorimeter, kcal/h
Q_e	Heat gained from electrical energy, kcal/h
Q_r	Heat generated by the residents, kcal/h
Q_s	Heat gained from solar radiation, kcal/h
Q_x	Average rate of heat derived from spurious heat sources, kcal/h
Q_y	Heat losses into ground from an individual zone of the uninsulated floor slab, kcal/h

ΔQ	Heat received in addition to that introduced by actual heating, kcal/ h
R	Radius of the circular slab including the lagging at its edges, m
R_o	Radius of the circular slab, excluding the lagging at its edges, m
$T(z), T(z,t)$	Temperature function representing the temperature distribution field inherent in the soil, ^oC
$T(x,y,z), T(r,z)$	Temperature function representing the stationary temperature distribution field produced in the soil by the slab, ^oC
$T(x,y,z,t)$	Temperature function representing the non-stationary temperature distribution field produced in the soil by the slab, ^oC
$T^*(\xi, z)$	Hankel's transform of the temperature function $T(r,z)$
T_o	Constant temperature; Mean temperature at heating tube level, ^oC
$T_o(t)$	Mean temperature at heating tube level, as a function of time, ^oC
T_1	Mean temperature in the measuring plane in the slab, below the heating tube level, ^oC
T_2	Mean temperature in the measuring plane in the slab, above the heating tube level, ^oC
T_a	Actual temperature at heating tube level in the floor slab, ^oC
T_s	Soil surface temperature, ^oC
V	Air space volume of the experimental house, m^3
a	Thermal diffusivity, m^2/h
a	Width of the slab, excluding the lagging at its edges, m
a'	Width of the slab including the lagging at its edges, m
b	Length of the slab, excluding the lagging at its edges, m
b'	Length of the slab including the lagging at its edges, m
c_p	Specific heat of air, at constant pressure, kcal/ kg ^oC
d	Thickness, cm
d_1	Distance of measuring plane T_1 from the heating tube level, T_o, cm
d_2	Distance of measuring plane T_2 from the heating tube level, T_o, cm
f	Relative humidity of air, %
k	Heat transmittance coefficient, kcal/ m^2 h ^oC
k_a	Apparent heat transmittance coefficient, kcal/ m^2 h ^oC
m	Thermal resistance, m^2 h ^oC/kcal
m_o	Thermal resistance of the soil under the uninsulated slab, when λ_o = 1 kcal/ m h ^oC, m^2 h ^oC/kcal
Δm	Thermal resistance of the additional lagging of the slab, m^2 h ^oC/ kcal
n	Number of air changes per hour; Integers 1, 2, 3, ...
p	Partial pressure of water vapour, kp/cm^2
p_s	Pressure of saturated water vapour, kp/cm^2
q_o	Density of upward heat flow from the ground, kcal/m^2 h
r	Variable circle radius in cylinder coordinates, m

r_o	Constant value of the variable circle radius, m
t	Time variable, h; days; Temperature, ^oC
t_i	Interior room temperature, ^oC
t_o	Outdoor air temperature, ^oC
x	Position coordinate, m
x_o	Constant value of position coordinate, m
y	Position coordinate, m
y_o	Constant value of position coordinate, m
z	Position coordinate; Depth, m
α	Parameter; Heat transfer coefficient, kcal/ m^2 h ^oC
β	Parameter
$\varkappa$	Coefficient reflecting the efficiency of thermal lagging
λ	Thermal conductivity, kcal/ m h ^oC
λ_o	Reference value of the thermal conductivity of soil, $\lambda_o = 1$ kcal/m h ^oC
μ	Factor defining the rate of temperature drop on the edge of the slab
ξ	Parameter
ϱ	Volume weight, kg/ m^3
τ	Time variable, h; days
φ	Moisture content of the soil, % of dry weight

INTRODUCTION

In building single-storey houses, one has recently adopted - mainly with a view to reducing the building costs - the practice of erecting such houses without basement, on a concrete slab laid immediately on the ground. It should be kept in mind that there are numerous instances when basement space, acquired at considerable cost, does not serve any useful purpose, while in other cases the cellarless solution is dictated by considerations of the available foundations. According to investigations carried out in the U.S.A. [1], the superterrestrial space of one-family houses may be increased by about 28 per cent at the expense of the basement.

There are numerous problems of heating technology and geotechnics associated with floor structures laid immediately on the ground. Most essential are: the thermal conditions in the soil under the slab, the heat losses from slab to ground, moisture conditions in the soil, and the risk of ground frost.

The object of the present work is to elucidate these questions theoretically and on the basis of the results derived from actual measurements. It is a fact that the conditions existing in the soil under buildings erected on a concrete slab are greatly variable, depending on the characteristics of the building site. In order to obtain a consistent idea of those conditions under the floor, of the heat losses through the floor to the ground and of other pertinent phenomena, it has therefore been considered appropriate to place considerable weight on the theoretical treatment of the subject matter in addition to the author's experimental investigations.

Questions associated with houses erected immediately on the ground have been rather extensively treated in generalized articles in various journals. Owing to their fairly close approximation of conditions prevailing in Finland, the articles by Asplund [2], Wredenfors [3] and Holmqvist [4], in which Swedish experience gained in actual practice and results of measurements are presented,

and Ericsson's [5] book "Hus utan källare; Grundläggningsmetoder" should be mentioned in this connection.

Henriksson [6] has investigated the soil temperature under unheated floors for different methods of floor insulation and the heat losses through the floor to the ground in experimental houses of a Swedish research institute (Statens forsknings-anstalt för lantmannabyggnader). He has also presented a method, based on the results of measurements, for calculating the heat losses through the floor, assuming the flow lines of the thermal flux to be circles. Jansson et al. [7] in the same research institute, have determined the heat losses to the ground through floors of the sheltering of cattle, and investigated the insulating questions of such floors.

Humbreys et al. [8], in Ohio, U.S.A., performed measurements of the soil temperature under heated floors with four different kinds of thermal insulation and determined the heat losses to the ground at different points of the floor slab with the aid of thermal flow meters.

Methods for calculation of the heat losses from a heated slab to the ground have also been presented by several other authors. The procedures suggested by Billington [9], Ruckli [10] and Krischer [11] have been briefly described in a previous publication of the present author [12]. Krischer's method has been further applied by Weyh [13] and by Rietschel and Reiss [14]. Among other things, the heat transfer coefficients of the floor surface and the external terrain as well as the heat transmittance factors of the snow cover have to be known when this method is applied. However, these quantities are not constant and they can be only inaccurately determined. Furthermore, the thermal conductivity of the soil has an unduly slight influence on the heat flow from the slab to the ground according to the said solutions.

Reports [15] on the results of investigations carried out in the U.S.A. state the experimentally determined heat losses for several kinds of floor structures, referred to unit length of the outer edge of the floor slab.

Scrutiny of the existing literature thus conveys the general impression that this extensive field of research has merely been broached so far and much work remains to be done in order to provide the designer with rules and data enabling him to make accurate predictions of the behaviour of floor slabs laid immediately on the ground in respect of their thermal properties and to choose serviceable and economic design solutions. This research should aim at establishment of a sound theory consistent with physical conditions encountered in actual practice; it should be sufficiently detailed and take into account the essential factors of influence; its results should be presented in a form adaptable without difficulty to calculations concerning floor slabs varying in size and in other respects.

1. THEORETICAL CALCULATIONS OF TEMPERATURE DISTRIBUTION FIELD AND HEAT FLOW

The transfer of heat to the soil from the foundation structure of a building erected immediately on the ground presents a three-dimensional problem of thermal conduction. Under the dimensional conditions concerned, the soil surface can be considered to be a plane upon which the heated slab is placed. The slab delivers heat by conduction into a semi-infinite medium, which has to be considered a homogeneous and isotropic substance of constant thermal diffusivity (a). The temperature distribution field existing under the heated floor slab results from superposition of the temperature field inherent in the soil, which is a given function of time and depth below soil surface, $T_1(z,t)$, and of the field produced in the soil by the action of the slab, this latter field being defined by a function of the position coordinates (x,y,z) and of time (t), $T_2(x,y,z,t)$.

Calculation of the temperature field under a building erected immediately upon the ground and of the heat flow from the slab into the soil requires knowledge of the temperature field at different depths in the undisturbed soil, which fluctuates periodically with the seasons, and of the phenomena of heat flow occurring in the soil. The author's investigations relating to these questions, both theoretical and based on experimental measurements, have been presented in a previous publication [16]. Furthermore, one has to know the thermal conductivity and thermal diffusivity (λ and a, respectively) of the soil, which depend on the soil type, the volume weight and the moisture content of the soil. For the purposes of the present work the thermal conductivities of the soil have been determined with the aid of data derived from Kersten's, Saare's and Wenner's measurements [17].

When calculating the temperature field produced in the soil by a heated slab, the soil temperature can be assumed to be 0 °C at any point, because the actual temperature distribution in the soil is separately accounted for. The equation for the stationary temperature field produced by the slab is obtained, under suitably

chosen boundary conditions, as the solution of a Laplacian differential equation. In the author's earlier investigation [12], the solutions giving the equations for the stationary temperature field and for the thermal flow have been presented for the cases of a slab having the shape of a rectangle, a narrow strip and a circle, with different boundary conditions. It could be observed on comparing the results with actual measurements that the boundary conditions best consistent with actual observations were those according to which the temperature of the slab is constant, T_o, over its entire area up to the thermal lagging at its edges. In the lagging at the edges the temperature declines linearly to 0 °C over the distance to the outer bordering edge of the building, where the lagging of the slab adjoins on snow. As a result of the heat dissipation from the edges of the slab there occurs melting of snow, and the temperature in the vicinity of this outer edge remains close to 0 °C through the greater part of the period during which the house is being heated, the heat flow produced by the slab in its nearest neighbourhood toward the surface being responsible for this condition. Also at greater distance from the wall the temperature at the surface remains close to 0 °C, as the house is surrounded by a fairly thick, well-insulating layer of snow [16].

The equations for a circular foundation slab under the said boundary conditions will be derived in the present work.

For a slab having the shape of a narrow strip, the equation of the temperature field was derived in the above-mentioned publication [12] also in the non-stationary case, when the temperature of the slab is a function of time. In order to render the calculations less complicated, simpler boundary conditions were employed in this instance, according to which the temperature on the edge of the slab falls from T_o to 0 °C by a step function. Physically, this boundary condition leads to an irrational result in that the heat flow at the edge of the slab becomes infinite; however, the error incurred in the temperature values in the central regions of the slab through the use of these boundary conditions is very small, less than 1 °C. The equation describing the field in the non-stationary case is introduced in this work for the purpose of comparisons between the stationary and non-stationary field.

11. THE INHERENT TEMPERATURE FIELD OF THE SOIL

The temperature distribution field $T_1(z, t)$ in free soil can be obtained as the solution of the one-dimensional differential equation

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial z^2} \tag{1}$$

when the thermal diffusivity $a = \frac{\lambda}{c\varrho}$ of the soil is known. Figs. 1, 2 and 3 show

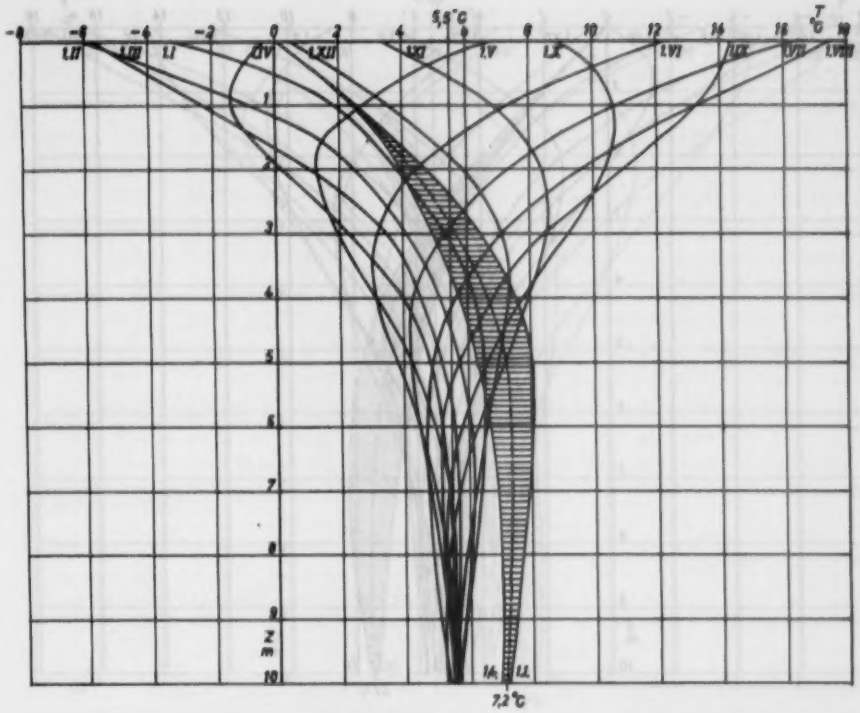

Fig. 1. Soil temperatures at different depths under bare ground on the first day of each month according to equation (1), and temperatures in snow-covered soil during the time January 1st – April 1st (shaded area). The dotted line represents the mean temperature distribution of the above-mentioned period. – Thermal diffusivity of the soil: $a = 0{,}0022\ m^2/h$.

the theoretically calculated temperatures at different depths in bare soil for the 1st day of each calendar month and for three different values of the soil's thermal diffusivity, namely, $a_1 = 0{,}0022\ m^2/h$, $a_2 = 0{,}0032\ m^2/h$, and $a_3 = 0{,}0047\ m^2/h$. The theoretical temperature curves were calculated, employing for the boundary conditions the mean value graph of the daily mean temperatures during the ten-year period 1945 – 1954 according to meteorological observations in Helsinki; they represent the mean temperatures during the said time in soil having no snow cover [16]. With increasing depth, the theoretically derived temperature curves approach asymptotically the mean temperature at soil surface of the said ten-year period, 5,5 °C. For instance, the temperatures in the soil (for $a = 0{,}0032\ m^2/h$) at 6 m depth deviate from this mean temperature by no more than ± 1 °C and those at 10 m depth only by ± 0,25 °C.

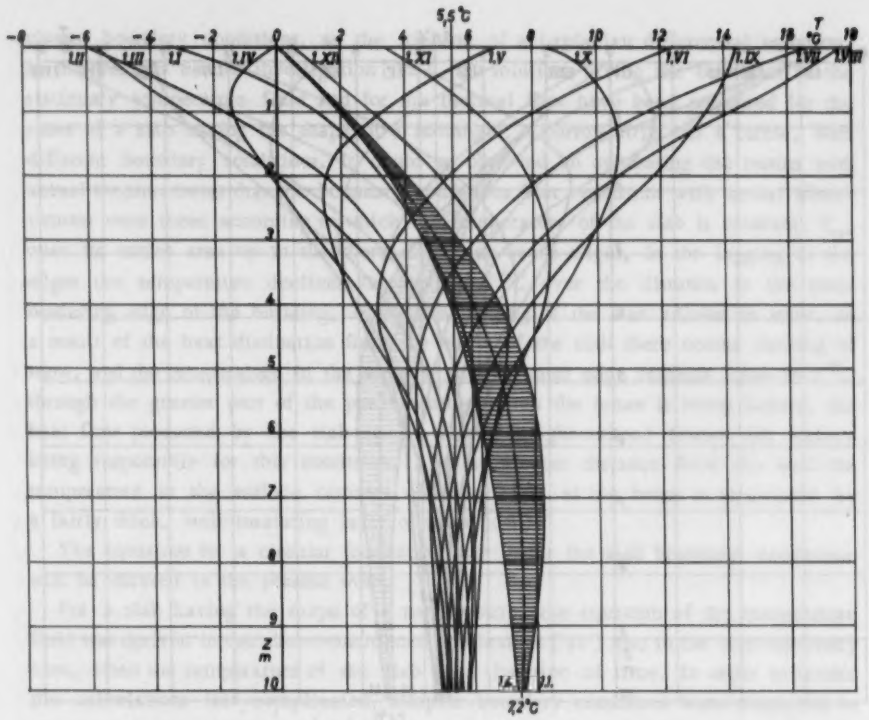

Fig. 2. Soil temperatures at different depths under bare ground on the first day of each month according to equation (1), and temperatures in snow-covered soil during the time January 1st – April 1st (shaded area). The dotted line represents the mean temperature distribution of the above-mentioned period. – Thermal diffusivity of the soil: $a = 0{,}0032\ m^2/h$.

Figs. 1, 2 and 3 also give the temperature distribution in snow-covered soil during the time from January 1st to April 1st (shaded area). The boundary curves have been calculated on the basis of the said ten-year mean with the aid of equation (9) of the above-mentioned publication [16], assuming the snow cover to have a thickness such that the soil surface temperature will not fall below 0 °C at any time during the cold season. The curve representing the mean of the curves for the 1st of January to the 1st of April can be considered the average temperature curve for the interior of the soil during the coldest interval of the heating period, when the soil lies under a thick snow cover as is the case in the conditions of Finnish climate.

The upper layers (between 1 and 2 m) of snow-covered soil have temperatures considerably in excess of those of bare soil. Under a fairly thick snow layer (40 to

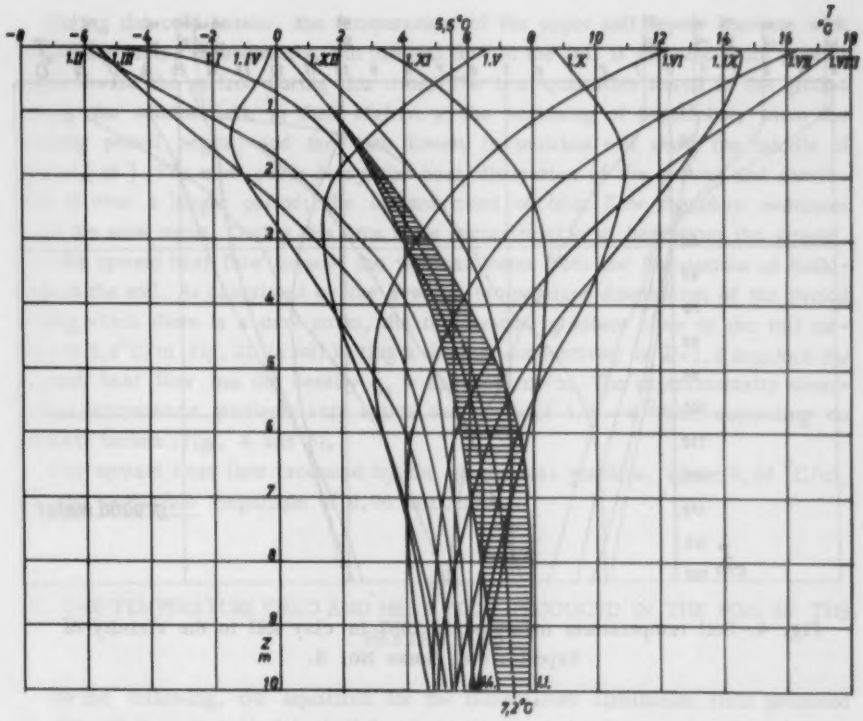

Fig. 3. Soil temperatures at different depths under bare ground on the first day of each month according to equation (1), and temperatures in snow-covered soil during the time January 1st ... April 1st (shaded area). The dotted line represents the mean temperature distribution of the above-mentioned period. – Thermal diffusivity of the soil: $a = 0,0047\ m^2/h$.

60 cm in thickness) the soil temperature remains approximately at 0 °C close to the surface (at 1 cm depth) even in midwinter [16]. Snow-covered soil has also a higher annual mean temperature than bare soil. Computing it from the mean value curve of the said ten-year period on assumption that the soil surface temperature under snow does not fall below 0 °C, the mean temperature in the soil is found to be 7,2 °C, that is, 1,7 °C higher than in bare soil. If accurate results are required when dealing with snow-covered soil, one has to use measured temperature values as the thermal insulating capacity of the snow cover varies and the soil surface temperature does not remain at exactly 0 °C under all circumstances. The actual temperatures of the soil measured in the neighbourhood of our experimental houses are presented in figures 4 and 5.

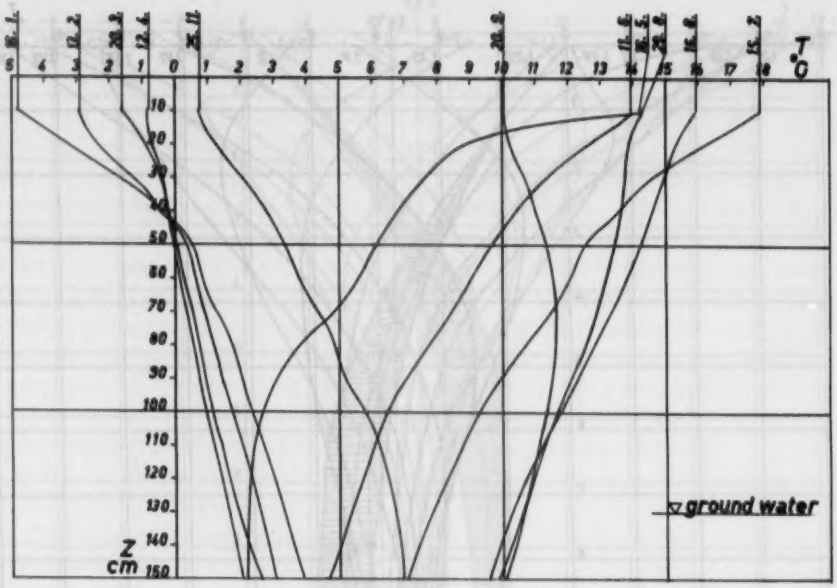

Fig. 4. Soil temperatures measured in 1957 in clay soil in the vicinity of Experimental House No. 3.

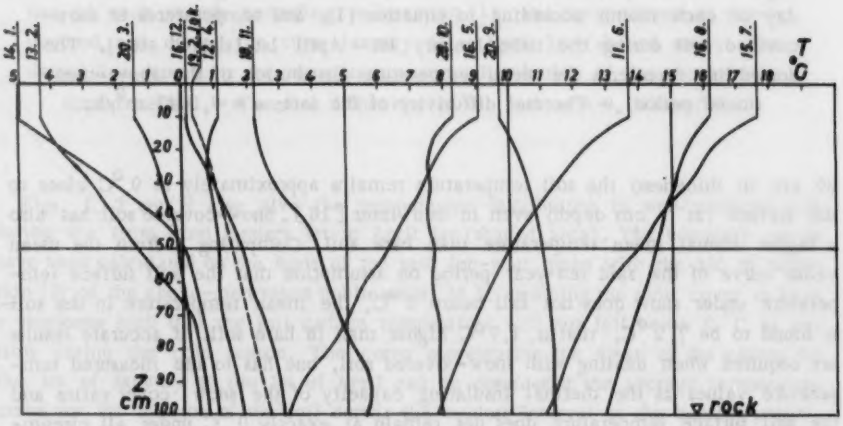

Fig. 5. Soil temperatures measured in 1957 in till soil in the vicinity of Experimental Houses No. 1 and 2, and temperature curve for soil under thick snow cover measured at Jokioinen on February 19th, 1959.

During the cold season, the temperatures of the upper soil layers increase with increasing depth; consequently, the thermal flow in the soil is directed from greater depth toward the surface during this time. The heat quantities stored in the ground during the summer are at their highest at the beginning of September, when the heating period begins, and they are lowest in snowless soil about the middle of March [16]. The snow cover delays the heat dissipation of the ground and distributes it over a longer period; the upward trend of heat flow therefore continues until the snow melts. During this time there occurs release of heat from the ground, and the upward heat flow reduces the thermal losses from the foundations of buildings to the soil. As calculated by the average temperature distribution of the period during which there is a snow cover, the temperature gradient close to the soil surface is 2,4°C/m (Fig. 2). In soil having a thermal conductivity of $\lambda = 1,5$ kcal/mh the upward heat flow has the density $q_o = 3,6$ kcal/m²h. The experimentally determined temperature gradients vary within the range of 1,5 ... 4 °C/m, depending on random factors (Figs. 4 and 5).

The upward heat flow produced by the geothermal gradient, about 0,03 °C/m, has the negligible magnitude of 0,06 kcal/m²h.

12. THE TEMPERATURE FIELD AND HEAT FLOW PRODUCED IN THE SOIL BY THE HEATED SLAB

In the following, the equations for the temperature distribution field produced in the soil by a heated slab shall be theoretically derived, assuming that the soil temperature is 0°C at any point. The field obtained in this manner will subsequently be combined by superposition with the temperature field inherent in the soil.

121. Stationary temperature field

1211. Rectangular foundation slab

The stationary temperature distribution field produced in the soil by a rectangular, heated slab and the thermal flow from the slab into the ground, which shall here be assumed to consist of a homogeneous and isotropic substance, have been calculated under the conditions set forth below and presented in an earlier work [12].

The function $T(x, y, z)$ describing the temperature field produced by the heated slab in the soil satisfies Laplace's equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \qquad (2)$$

and the boundary conditions

$$
\begin{aligned}
&\text{a)}\ T(x, y, 0) = T_o && \text{when } -a \leq x \leq a \text{ and } -b \leq y \leq b \\
&\text{b)}\ T(x, y, 0) = T_o - \frac{(x-a)\,T_o}{a'-a} && \text{"} \quad a < x < a' \\
&\text{c)}\ T(x, y, 0) = T_o - \frac{(y-b)\,T_o}{b'-b} && \text{"} \quad b < y < b' \qquad (3) \\
&\text{d)}\ T(x, y, 0) = 0 && \text{"} \quad x \geq a' \\
&\text{e)}\ T(x, y, 0) = 0 && \text{"} \quad y \geq b' \\
&\text{f)}\ T(x, y, z) \to 0 && \text{"} \quad x \to \infty,\ y \to \infty \text{ and } z \to \infty
\end{aligned}
$$

$-\frac{T_o}{a'-a}$ and $-\frac{T_o}{b'-b}$ characterize the rate at which the temperature falls in the x and y directions, respectively, at the edges of the slab. It is moreover assumed that $\frac{a'-a}{a} = \frac{b'-b}{b}$. This condition had to be imposed in order to obtain for the temperature function $T(x, y, z)$ a form sufficiently simple for the subsequent numerical calculations.

Boundary condition (3)a states that the temperature of the slab is constant, T_o, independent of the position coordinates. Boundary conditions (3)b and (3)c imply that the temperature falls at the lagged edges of the slab at a linear rate from T_o to 0 °C, over the distance $a'-a$ in the direction of the x axis and over the distance $b'-b$ in the direction of the y axis. For a square slab, $a'-a = b'-b$, while for a rectangular slab the interval $a'-a$ is smaller than $b'-b$ when a is the smaller side of the rectangle. This means that the temperature declines at a slightly lesser rate at the short ends than on the long sides of the building. If the course of the temperature function in the region covered by the lagging at the edges is chosen in numerical calculations so that it conforms with the measurements in the x direction, the resulting values for the temperature distribution in homogeneous soil in cross sections under the central part of the slab will be correct with a high degree of accuracy. The same procedure can also be applied to longitudinal sections.

The temperature function has been derived with the aid of Fourier's double transformations; it is

$$T(x, y, z) = \frac{2T_o}{\pi^2} \frac{a}{(a'-a)} \int_0^\infty \int_0^\infty \left[\frac{\sin \frac{a'}{a}(\alpha-\beta) - \sin(\alpha-\beta)}{(\alpha-\beta)\alpha\beta} - \frac{\sin \frac{a'}{a}(\alpha+\beta) - \sin(\alpha+\beta)}{(\alpha+\beta)\alpha\beta} \right]$$

$$e^{-z\sqrt{\left(\frac{\alpha}{a}\right)^2 + \left(\frac{\beta}{b}\right)^2}} \cos\left(\frac{x}{a}\alpha\right) \cos\left(\frac{y}{b}\beta\right) d\alpha d\beta \qquad (4)$$

where α and β are parameters [12].

The temperature field. The temperature field in the soil under a rectangular slab, calculated by equation (4), has been shown in Figures 6, 7 and 8 for vertical sections through the axes and the diagonal of the slab.

The slab has the dimensions a = 4,0 m, b = 6,2 m and the temperature T_o = 30 °C. The temperature falls linearly from T_o to 0 °C over the distance a'- a = 0,50 m in the direction of the x axis and over the distance b'- b ≈ 0,78 m in that of the y axis.

The distance a'- a = 0,50 m, over which the temperature falls from T_o to 0 °C, is made up of three different intervals, according to actual measurements, each one contributing its own share to the temperature drop. They are: a traverse of the slab itself, about 10 cm in width; the edge-lagging 30 cm in thickness; and a distance of about 10 cm outward from this lagging. The temperature falls less abruptly at the short ends of the building.

Heat flow. The thermal flow from the area $|x| \leq x_o$, $|y| \leq y_o$ of a rectangular, heated slab into homogeneous soil is calculated by the following equation [12]:

$$Q = \frac{8\lambda T_o a^2 b}{\pi^2 (a'-a)} \int_0^\infty \int_0^\infty \left[\frac{\sin \frac{a'}{a}(\alpha-\beta) - \sin(\alpha-\beta)}{(\alpha-\beta)\alpha\beta} - \frac{\sin \frac{a'}{a}(\alpha+\beta) - \sin(\alpha+\beta)}{(\alpha+\beta)\alpha\beta} \right]$$

$$\sqrt{\left(\frac{\alpha}{a}\right)^2 + \left(\frac{\beta}{b}\right)^2} \frac{\sin\left(\frac{x_o}{a}\alpha\right) \sin\left(\frac{y_o}{b}\beta\right)}{\alpha\beta} d\alpha d\beta \qquad (5)$$

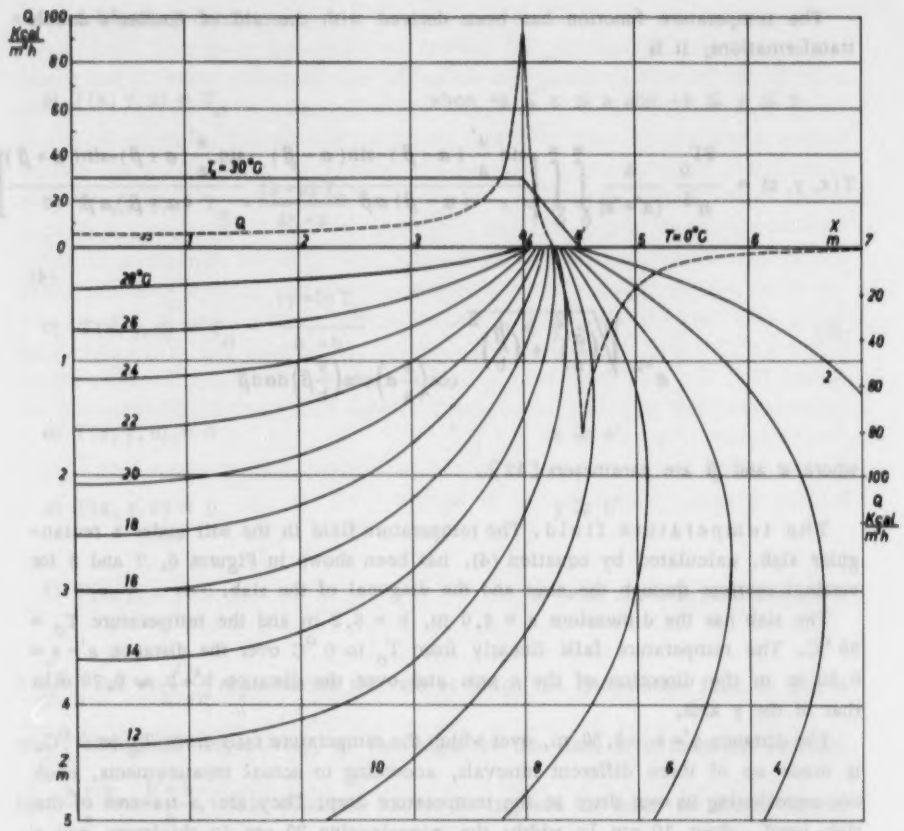

Fig. 6. Stationary temperature distribution field produced in the soil by a rectangular slab (a = 4,0 m, b = 6,2 m; a'- a = 0,50 m, b'- b = 0,78 m) and thermal flow from the slab into ground in a cross section through the centre of the slab according to equation (4) when T_o = 30 °C and λ_o = 1 kcal/m h °C.

In the region

$$x_o \leq a$$

$$y_o \leq b$$

the direction of the heat flow is perpendicularly downward from the surface of the slab. For the heat flow supplied by the entire slab, the values of x_o and y_o con-

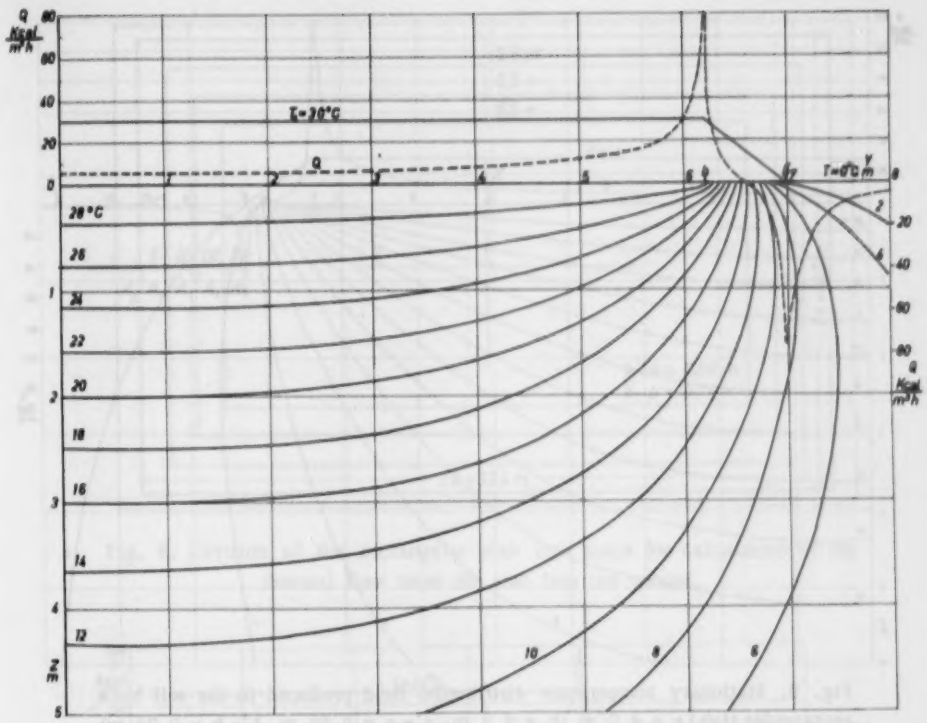

Fig. 7. Stationary temperature distribution field produced in the soil by a rectangular slab (a = 4,0 m, b = 6,2 m; a'- a = 0,50 m, b'- b = 0,78 m) and thermal flow from the slab into ground in a longitudinal section through the centre of the slab according to equation (4) when T_o = 30 °C and λ_o = 1 kcal/m h °C.

sistent with the sign of equality are used. The heat flow for individual zones (Fig. 9) is obtained by determining the difference of the heat flows delivered by two rectangular areas.

Under the boundary conditions (3), physically acceptable solutions are obtained both for the temperature field and the thermal flow.

1212. Foundation slab having the shape of a narrow strip

In the vertical section of a slab having the shape of a narrow strip, two-dimensional equations are valid which are derived from equations (4) and (5) by the

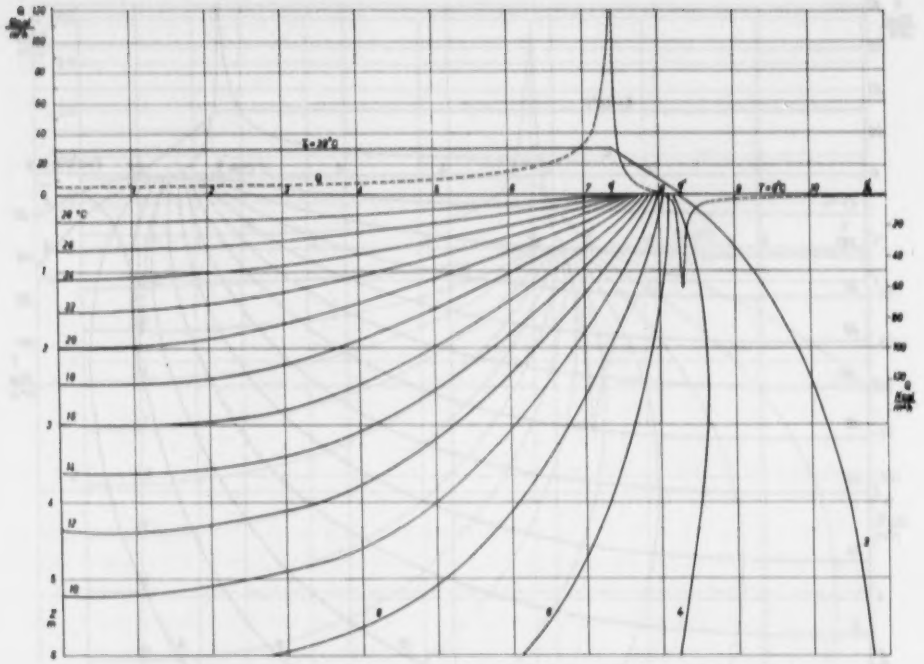

Fig. 8. Stationary temperature distribution field produced in the soil by a rectangular slab (a = 4,0 m, b = 6,2 m; a'- a = 0,50 m, b'- b = 0,78 m) and thermal flow from the slab into ground in a section through the diagonal of the slab according to equation (4) when T_o = 30 °C and λ_o = 1 kcal/m h °C.

limit process b → ∞ [12]. The temperature distribution field is described by the equation

$$T(x, z) = \frac{2T_o\, a}{\pi(a-a')} \int_0^\infty \frac{\cos\frac{a'}{a}\alpha - \cos\alpha}{\alpha^2} \cos\left(\frac{x}{a}\alpha\right) e^{-\frac{z}{a}\alpha}\, d\alpha \qquad (6)$$

The heat flow that maintains in the soil under a slab having the shape of a narrow strip a temperature field conforming to equation (6) is calculated zone by zone from the equation

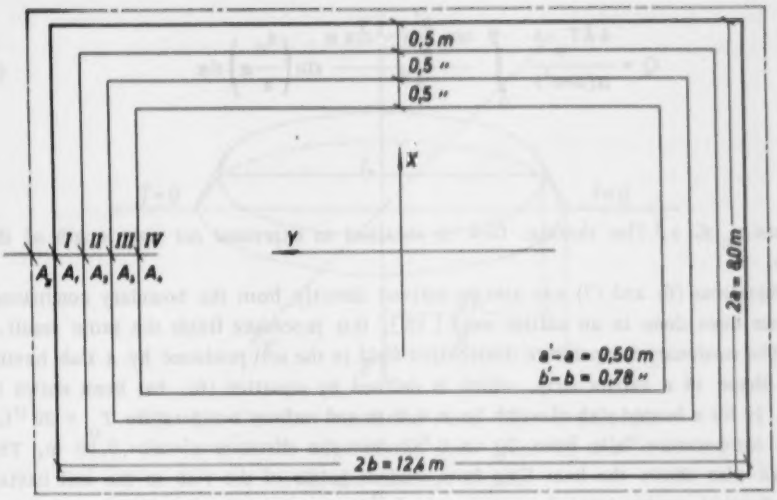

Fig. 9. Division of the rectangular slab into zones for calculation of the thermal flow from the slab into the ground.

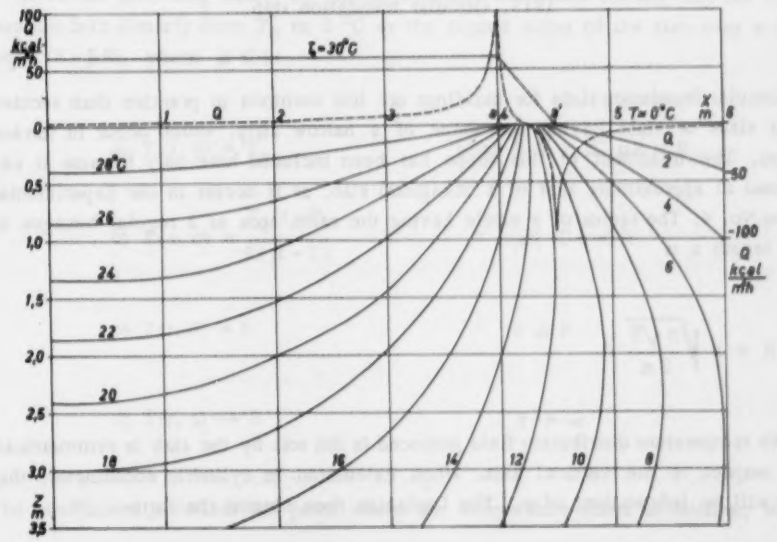

Fig. 10. Stationary temperature distribution field produced in the soil by a slab having the shape of a narrow strip (a = 4,0 m, a'- a = 0,50 m) and thermal flow from the slab into ground according to equation (6) when T_0 = 30°C and λ_0 = 1 kcal/m h °C.

$$Q = \frac{4\lambda T_0 a}{\pi(a-a')} \int_0^{\infty} \frac{\cos\frac{a'}{a}\alpha - \cos\alpha}{\alpha^2} \sin\left(\frac{x_0}{a}\alpha\right) d\alpha \qquad (7)$$

where $x_0 \leq a$. The thermal flow is obtained as expressed per unit length of the slab.

Equations (6) and (7) can also be derived directly from the boundary conditions, as has been done in an earlier work [12]; this procedure fields the same result.

The stationary temperature distribution field in the soil produced by a slab having the shape of a narrow strip, which is defined by equation (6), has been shown in Fig. 10 for a heated slab of width 2a = 8,0 m and surface temperature T_0 = 30 °C. The temperature falls from T_0 to 0 °C over the distance a'- a = 0,50 m. The figure also shows the heat flow from various points of the slab to the soil having the thermal conductivity λ_0 = 1 kcal/m h °C.

1213. Circular foundation slab

Circular foundation slabs for buildings are less common in practice than rectangular slabs or slabs having the shape of a narrow strip, which occur in terrace houses. The treatment of this shape has been included here only because it can be used to approximate that of a hexagonal slab, as it occurs in our Experimental House No. 4. The radius of a circle having the same area as a regular hexagon of side length a is

$$R = a\sqrt{\frac{3\sqrt{3}}{2\pi}}$$

The temperature distribution field produced in the soil by the slab is symmetrical with respect to the vertical axis. When calculated in cylindric coordinates, the field will be independent of φ. The Laplacian then obtains the form

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0 \qquad (8)$$

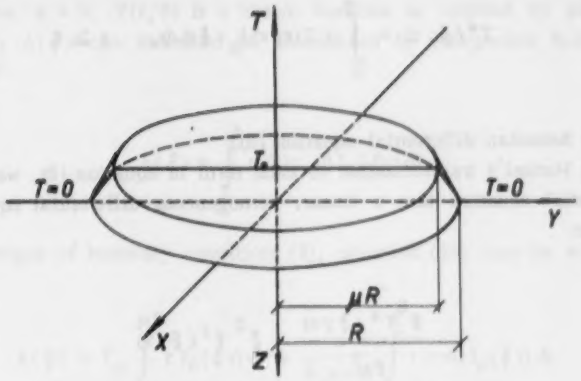

Fig. 11. Circular foundation slab. The temperature decreases at the edge of the slab from T_0 to 0 °C over the distance $R-\mu R$.

and the function $T(r, z)$ describing the temperature field satisfies this equation.

As in the preceding cases, we choose boundary conditions stating that the temperature falls linearly from T_0 to 0 °C at the lagged edges of the slab over a distance $(R-\mu R)$, where $\mu < 1$:

$$
\begin{aligned}
&\text{a) } T(r, 0) = T_0 && \text{when } 0 \leq r \leq \mu R = R_0\\
&\text{b) } T(r, 0) = \frac{T_0(r-R)}{R(\mu-1)} && \text{"} \quad \mu R < r < R\\
&\text{c) } T(r, 0) = 0 && \text{"} \quad r \geq R\\
&\text{d) } T(r, z) \to 0 && \text{"} \quad z \to \infty
\end{aligned}
\tag{9}
$$

The coefficient μ defines the point where the temperature starts to decline, while $\frac{T_0}{R(\mu-1)}$ characterizes the rate of descent (Fig. 11).

Hankel's transformation of the order 0 of the temperature function $T(r, z)$

$$T^*(\xi, z) = \int_0^\infty r\, T(r, z) J_0(\xi r)\, dr, \qquad z \geq 0 \tag{10}$$

satisfies the Besselian differential equation (8).

Applying Hankel's transformation to each term in equation (8), we can convert the differential equation into a linear, homogeneous differential equation of the second order:

$$\frac{\partial^2 T^*(\xi, z)}{\partial z^2} = \xi^2 T^*(\xi, z) \tag{11}$$

We see that Hankel's transformation of the temperature distribution function satisfies equation (11).

The equation has the general solution

$$T^*(\xi, z) = A(\xi)\, e^{-\xi z} + B(\xi) e^{+\xi z} \tag{12}$$

Complying with boundary condition (9)d, we chose the finite branch of the function. Therefore, $B(\xi) \equiv 0$ and

$$T^*(\xi, z) = A(\xi)\, e^{-\xi z} \tag{13}$$

The unknown function $A(\xi)$ occurring in this equation is determined in compliance with the boundary conditions. We obtain from equations (10) and (13):

$$\int_0^\infty r\, T(r, z) J_0(\xi r)\, dr = A(\xi)\, e^{-\xi z}, \quad z \geq 0 \tag{14}$$

According to Hankel's inversion theorem, the temperature distribution function satisfying equation (8) is [18]:

$$T(r, z) = \int_0^\infty \xi\, A(\xi) J_0(\xi r)\, e^{-\xi z}\, d\xi \tag{15}$$

On the surface, $z = 0$, $T(r, 0)$ is a known function as implied by the boundary conditions (9). $A(\xi)$ can therefore be determined by integration from equation (14) for $z = 0$:

$$A(\xi) = \int_0^\infty r\, T(r, 0)\, J_0(\xi r)\, dr \tag{16}$$

On the strength of boundary conditions (9), equation (16) may be written

$$A(\xi) = T_0 \int_0^{\mu R} r\, J_0(\xi r)\, dr + \frac{T_0}{R(\mu - 1)} \int_{\mu R}^{R} r(r - R)\, J_0(\xi r)\, dr \tag{17}$$

$$A(\xi) = \frac{T_0}{(\mu - 1)} \left[\frac{1}{\xi^2} J_0(\xi R) - \frac{\mu}{\xi^2} J_0(\xi \mu R) - \frac{1}{R\xi^2} \int_{\mu R}^{R} J_0(\xi r)\, dr \right] \tag{18}$$

Substituting the expression (18) for $A(\xi)$ in equation (15), we find the temperature distribution function

$$T(r, z) = \frac{T_0}{\mu - 1} \int_0^\infty \left[\frac{J_0(\xi R) - \mu J_0(\xi \mu R)}{\xi} - \frac{1}{R\xi} \int_{\mu R}^{R} J_0(\xi r)\, dr \right] J_0(\xi r)\, e^{-\xi z}\, d\xi \tag{19}$$

For numerical calculations it is advantageous to convert equation (19) into a more suitable form by substituting $\xi R = \alpha$:

$$T(r, z) = \frac{T_0}{\mu - 1} \int_0^\infty \left[\frac{J_0(\alpha) - \mu J_0(\mu\alpha)}{\alpha} - \frac{1}{R\alpha} \int_{\mu R}^{R} J_0\left(\frac{r}{R}\alpha\right) dr \right] J_0\left(\frac{r}{R}\alpha\right) e^{-\frac{z}{R}\alpha}\, d\alpha \tag{20}$$

in which the expression in angular brackets can be written as a series:

$$\left[\ \right] = \sum_{n=1}^{\infty} \frac{(-1)^n\, n\left(1 - \mu^{2n+1}\right)}{(2n+1)\,(n!)^2\, 2^{2n-1}}\, \alpha^{2n-1} \tag{21}$$

When $\mu \to 1$ in equation (20), the temperature distribution function $T(r, z)$ obtains the limiting value

$$T(r, z) = T_0 \int_0^\infty J_1(\alpha) J_0\left(\frac{r}{R}\alpha\right) e^{-\frac{z}{R}\alpha} \, d\alpha \qquad (22)$$

because

$$\lim_{\mu \to 1} \frac{1}{\mu - 1} \sum_{n=1}^{\infty} \frac{(-1)^n n\left(1-\mu^{2n+1}\right)}{(2n+1)(n!)^2 2^{2n-1}} \alpha^{2n-1} = J_1(\alpha)$$

Equation (22) can also be derived directly from the boundary conditions (9), which assume the following form for $\mu = 1$:

a) $T(r, 0) = T_0$ when $0 \leq r < R$
b) $T(r, 0) = 0$ " $r > R$
c) $T(r, z) \to 0$ " $z \to \infty$

The expression for the heat flow becomes infinite for these boundary conditions, which therefore give a physically irrational solution [12].

Heat flow. A circular slab having the radius $R_0 = \mu R$ and the temperature T_0 delivers to the soil, at the distance r from its centre, the thermal flow per unit area:

$$Q(r) = -\lambda \frac{\partial T(r, 0)}{\partial z}$$

$$Q(r) = \frac{\lambda T_0}{R(\mu - 1)} \int_0^\infty \left[\mu \, J_0(\mu\alpha) - J_0(\alpha) + \frac{1}{R} \int_{\mu R}^{R} J_0\left(\frac{r}{R}\alpha\right) dr \right] J_0\left(\frac{r}{R}\alpha\right) d\alpha \qquad (23)$$

The heat flow supplied by the area $r \leq r_0$ of the slab is obtained from the equation

$$Q = \frac{2\pi r_0 \lambda T_0}{\mu - 1} \int_0^\infty \left[\frac{\mu J_0(\mu\alpha) - J_0(\alpha)}{\alpha} + \frac{1}{R\alpha} \int_{\mu R}^{R} J_0\left(\frac{r}{R}\alpha\right) dr \right] J_1\left(\frac{r_0}{R}\alpha\right) d\alpha \qquad (24)$$

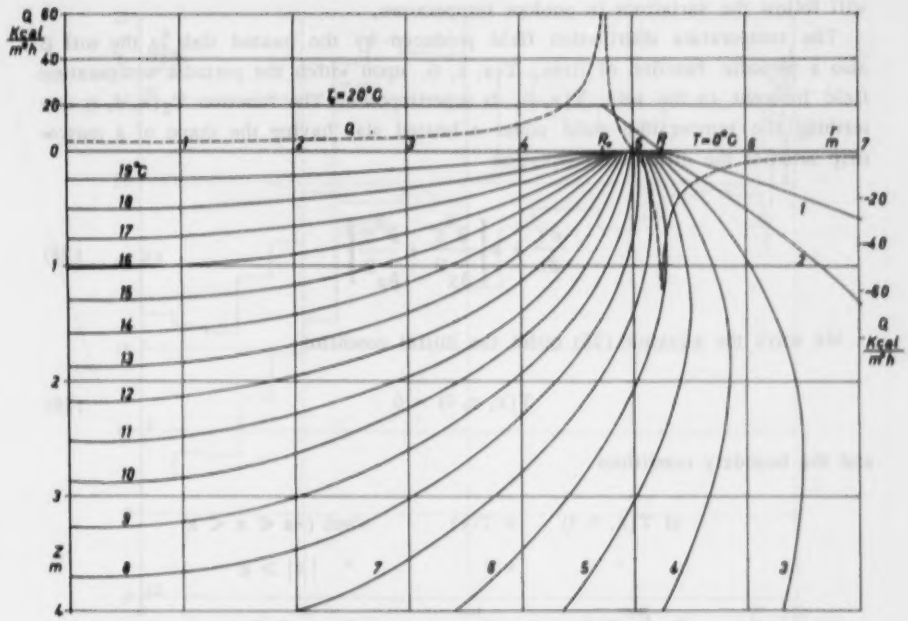

Fig. 12. Stationary temperature distribution field produced in the soil by a circular slab (R_0 = 4,73 m, R = 5,26 m, μ = 0,9) and thermal flow from the slab into ground according to equations (20) and (23) when T_0 = 20 °C and λ_0 = 1 kcal/m h °C.

The heat flow from different annular zones of the slab is obtained by giving different values to r_0. For $r_0 = R_0$ this procedure yields the total heat flow supplied by the slab.

Fig. 12 shows the temperature distribution field described by equation (20) and the heat flow calculated from (23).

122. Non-stationary temperature field; foundation slab having the shape of a narrow strip

In the preceding cases the temperature of the heated slab was assumed to be constant and the temperature distribution field under steady-state conditions was studied. However, the temperature of the slab will vary during the heating period according to the heat requirements of the house, depending on the season, and it

will follow the variations in outdoor temperature.

The temperature distribution field produced by the heated slab in the soil is also a periodic function of time, $T(x, z, t)$, upon which the periodic temperature field inherent in the soil, $T(z, t)$, is superimposed. The function $T_2(x, z, t)$ describing the temperature field under a heated slab having the shape of a narrow strip satisfies the differential equation

$$\frac{\partial T}{\partial t} = a\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right] \tag{25}$$

We solve the equation (25) under the initial condition

$$T(x, z, 0) = 0 \tag{26}$$

and the boundary conditions

$$\begin{aligned}
&\text{a)}\quad T(x, 0, t) = T(t) && \text{when } (-a < x < a)\\
&\qquad\quad " \quad = 0 && " \quad |x| > a\\
&\text{b)}\quad \frac{\partial}{\partial x} T(0, z, t) = 0 && " \quad z \geq 0\\
&\text{c)}\quad T(x, z, t) \to 0 && \text{when } \begin{cases} x \to \infty \\ z \to \infty \end{cases}
\end{aligned} \tag{27}$$

The soil is assumed to consist of a homogeneous, isotropic substance of constant thermal diffusivity a.

The equation has been solved in an earlier work [12] with the aid of Fourier's sine and cosine transformations, obtaining for the temperature field the equation

$$T(x, z, t) = \frac{a}{4\sqrt{\pi}} \int_0^t \frac{z\, e^{-\frac{z^2}{4a(t-\tau)}}}{[a(t-\tau)]^{3/2}} \left[\operatorname{erf}\frac{x+a}{\sqrt{4a(t-\tau)}} - \operatorname{erf}\frac{x-a}{\sqrt{4a(t-\tau)}}\right] T(\tau)\, d\tau \tag{28}$$

which satisfies equation (25) and the boundary conditions (26) and (27).

Equation (28) describes the temperature produced by the slab in the point (x, z) at the time t when $z > 0$. When the variation of the temperature of the slab is known as a function of time, $T(\tau)$ (Fig. 13), the temperature field is found by means of equation (28) (Fig. 14).

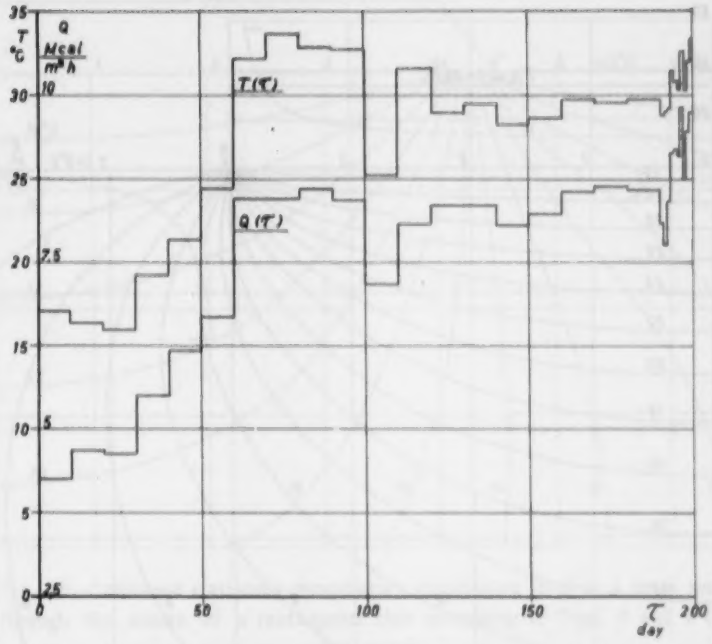

Fig. 13. Mean over-temperature $T(\tau)$ of the heating tube level over the ground surface and variation of the heat flow from the slab into ground as a function of time in Experimental House No. 1 during the heating period 1956 – 1957 according to performed measurements.

The heat flow plotted in Fig. 14 has been calculated from the isotherm graph representing the temperature distribution field.

13. SUPERPOSITIONING THE TEMPERATURE FIELD INHERENT IN THE SOIL ON THE FIELD PRODUCED BY THE HEATED SLAB

The temperature distribution field inherent in the soil $T_1(z, t)$, as presented in Figs. 1, 2 and 3 on the basis of theoretical calculations, or as found by means of measurements, will be combined with the field produced by the slab, $T_2(x, y, z, t)$, by the procedure of superposition. This yields the actual temperature distribution field under the slab and in its immediate surroundings

$$T(x, y, z, t) = T_1(z, t) + T_2(x, y, z, t) \tag{29}$$

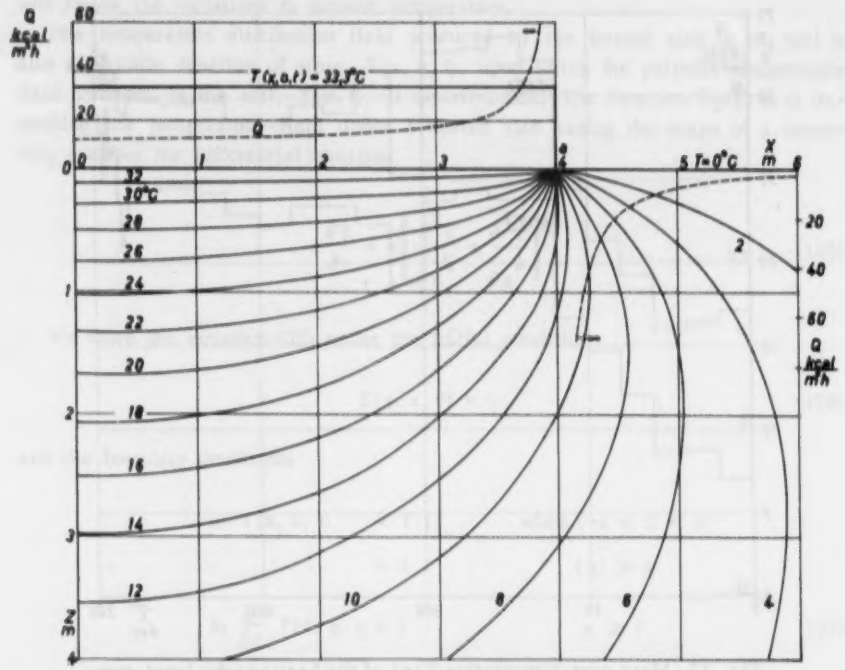

Fig. 14. Non-stationary temperature distribution field produced in the soil by a slab having the shape of a narrow strip (a = 4,0 m) at the time t = 200 days, (March 20th; zero time: September 1st) according to equation (28), and thermal flow from the slab into ground when λ_o = 1 kcal/m h °C.

The process of superposition consists in drawing, for instance for a rectangular slab, the curves $T = T(x', y', z)$ for different pairs of values (x', y'). The values read from the temperature curves obtained in this manner are then augmented by the soil temperature at different depths, resulting in the temperature distribution as a function of depth for each pair of values (x', y'). These new temperature curves can be evaluated by plotting the isothermal lines of the combined field.

In actual practice, however, the true surface temperature of the slab, T_a, is known from measurements. T_a corresponds to a temperature value obtained by superposition, i.e., the sum of the slab temperature and the soil surface temperature, $T_a = T_o + T_s$. In the superpositioning process, the field of the slab is calculated, putting for the surface temperature of the slab its temperature excess T_o over the soil surface, $T_o = T_a - T_s$.

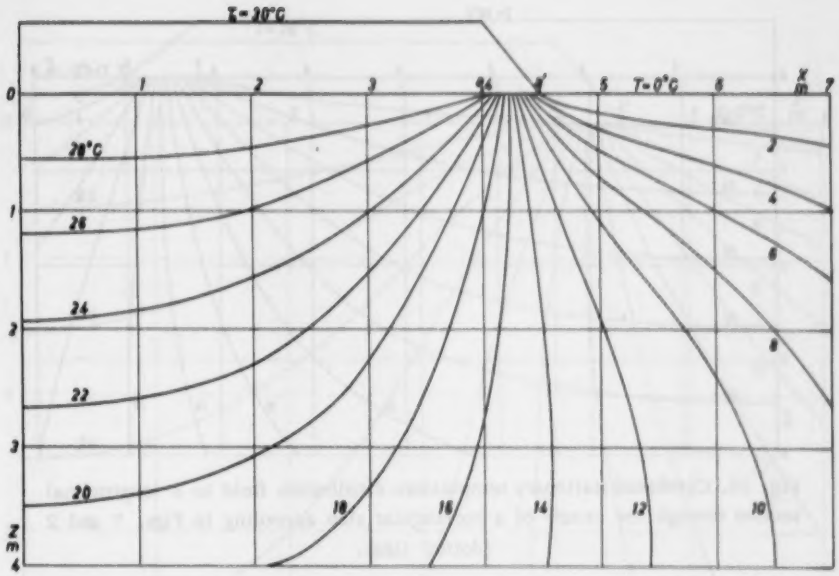

Fig. 15. Combined stationary temperature distribution field in a cross section through the centre of a rectangular slab according to Figs. 6 and 2 (dotted curve).

For a rectangular slab (with a = 4,0 m, b = 6,2 m), the theoretically calculated, stationary temperature field (without contribution of the temperature field inherent in the soil) has been shown in Figs. 6, 7 and 8. We obtain the field consistent with actual conditions by superimposing upon that of Figs. 6, 7 and 8 the theoretically calculated, average temperature distribution in snow-covered soil (Fig. 2). This is done in the manner described above. Figs. 15, 16 and 17 show the resulting, combined temperature distribution field in the planes corresponding to transversal, longitudinal and diagonal sections of the slab. They illustrate the average conditions in homogeneous soil under an uninsulated slab when the surroundings of the slab have a snow cover of sufficient thickness to ensure maintenance of a soil surface temperature of 0 °C.

For a slab having the shape of a narrow strip, the combined temperature distribution field is obtained by superimposing the field produced by the slab (Fig. 10) on the theoretically calculated, average temperature distribution field of snow-covered ground (Fig. 2). Fig. 18 shows the combined field resulting from this computation.

For a circular slab, the actual temperature distribution field existing under it will be obtained by superposition of the theoretically calculated, stationary tem-

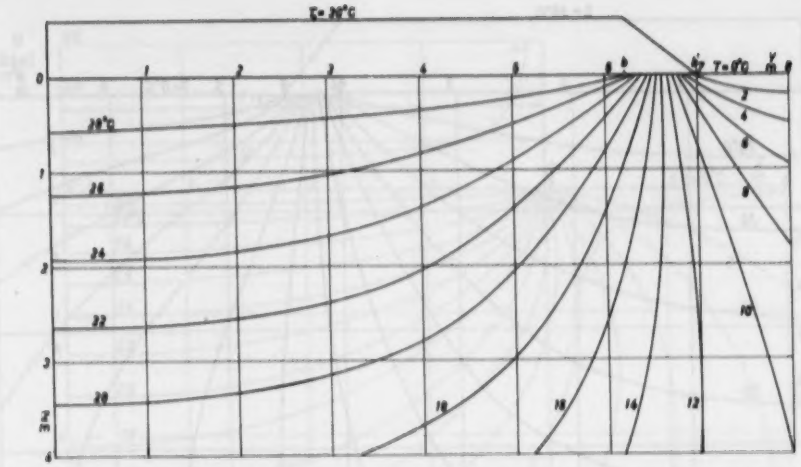

Fig. 16. Combined stationary temperature distribution field in a longitudinal section through the centre of a rectangular slab according to Figs. 7 and 2 (dotted line).

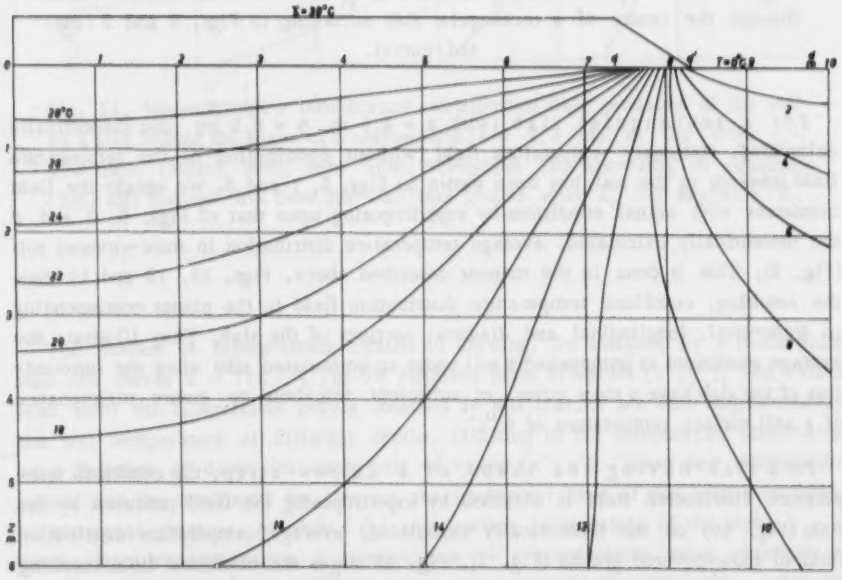

Fig. 17. Combined stationary temperature distribution field in a section through the diagonal of a rectangular slab according to Figs. 8 and 2 (dotted line).

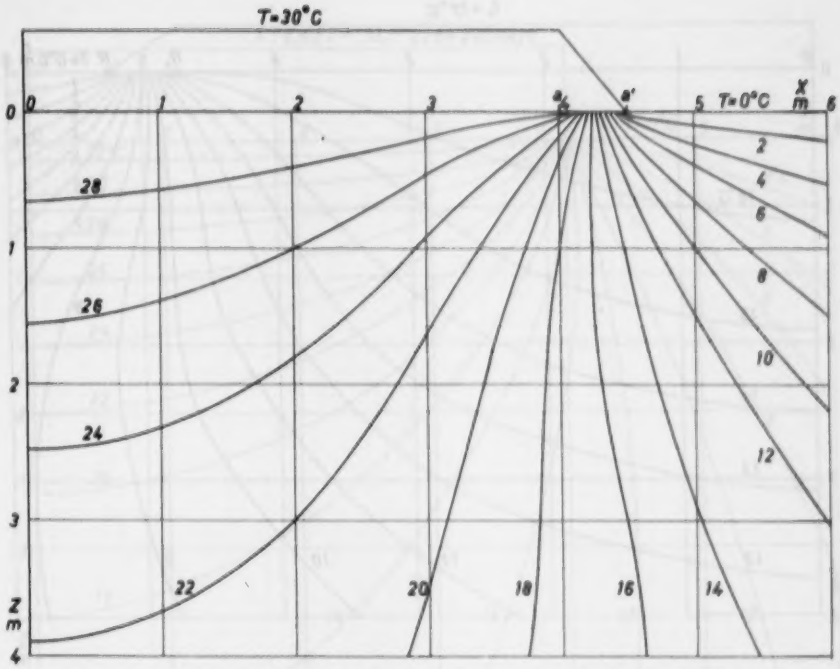

Fig. 18. Combined stationary temperature distribution field for a slab having the shape of a narrow strip according to Figs. 10 and 2 (dotted line).

perature field modified from Fig. 12, under such a slab, and the theoretically calculated, average temperature distribution field of snow-covered ground (Fig. 2). In this manner, the combined field shown in Fig. 19 has been found. The particular slab to which this calculation refers, has the radius R_o = 4,73 m, its area being equal to that of the hexagonal floor slab in Experimental House No. 4; the temperature drop at its edge has been assumed to occur over the distance R - μR = 0,526 m (cf. Fig. 27). The surface temperature chosen as a basis for the calculations, T_o = 17 °C, is consistent with the average surface temperature of the unheated slab in the said instance of actual practice.

For the non-stationary case, a calculation has been carried out, superimposing upon each other the temperature field existing on the 20th of March in soil having a thermal diffusivity of a = 0,0032 m^2/h, under a slab having the shape of a narrow strip, (Fig. 14) and the simultaneous temperature field in snow-covered

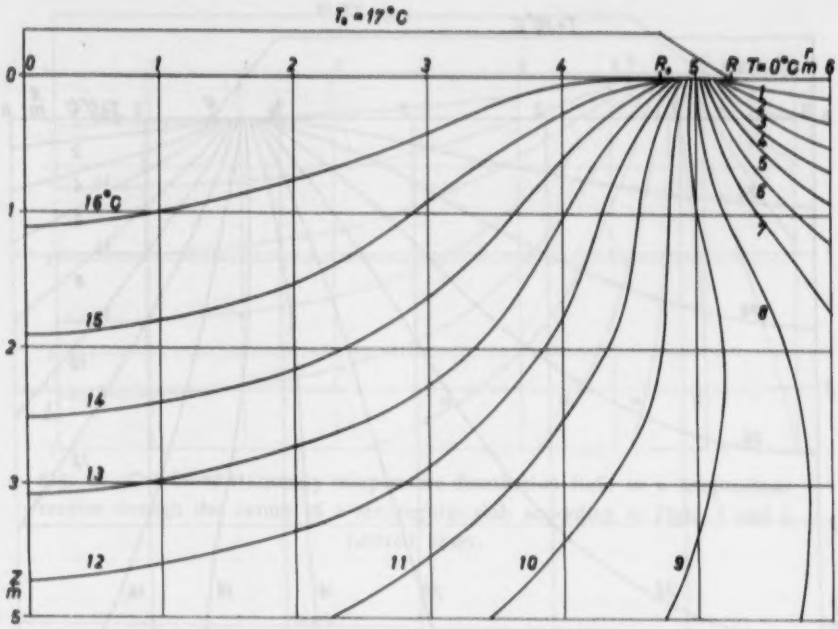

Fig. 19. Combined stationary temperature distribution field for a circular slab (R_o = 4,73 m, R = 5,26 m) according to Fig. 12 (modified for surface temperature T_o = 17 °C) and Fig. 2 (dotted line).

soil Fig. 2. The combined field in this instance is shown in Fig. 20. At the edges of the slab, where the isothermal lines are closely spaced, this field is inconsistent with actual conditions, owing to the boundary conditions that were used in deriving equation (28) and in plotting the curves of Fig. 14.

14. DISCUSSION OF THE THEORETICAL RESULTS

If the equations of the non-stationary temperature distribution are solved for more complicated boundary conditions than those set forth in equations (27), the resulting equations are too cumbersome. Also in the case of the two-dimensional equation (28) derived from the boundary conditions (27) the calculation of temperatures at different points and for different times presents a great amount of work

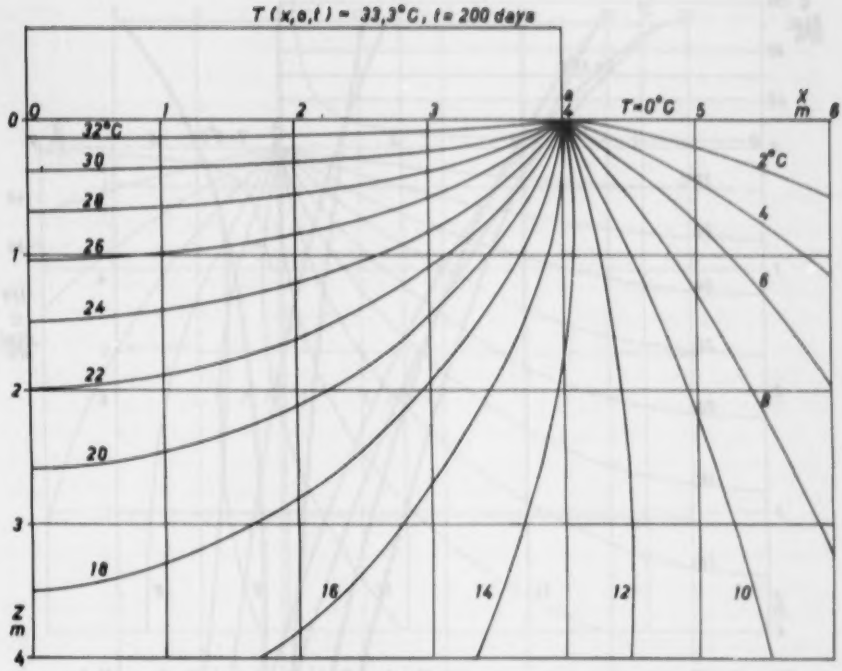

Fig. 20. Non-stationary temperature distribution field produced in the soil (a = 0,0032 m^2/h) by a slab having the shape of a narrow strip, the field of the slab (Fig. 14) being combined with the theoretical simultaneous field in snow-covered ground (Fig. 2 dotted line).

because the function $T(\tau)$ representing the surface temperature of the slab appears under the integral sign and numerical integration is necessary with respect to τ in all cases. With stationary fields, on the other hand, it is possible to pass from one slab temperature to another merely by multiplication by a ratio figure.

The question whether there is sufficient justification to use for the calculation of the temperature field produced in the soil by a heated slab the stationary instead of the non-stationary solution, can be decided by comparing the stationary and non-stationary fields calculated for a slab having the shape of a narrow strip.

Fig. 14 gave a representation of the temperature field in the soil (having a = 0,0032 m^2/h) under different points of a slab having the shape of a narrow strip (with a = 4,0 m), calculated by equation (28) for the time t = 200 days (t = 0 corresponding to the date September 1st, 1956) when the surface temperature of

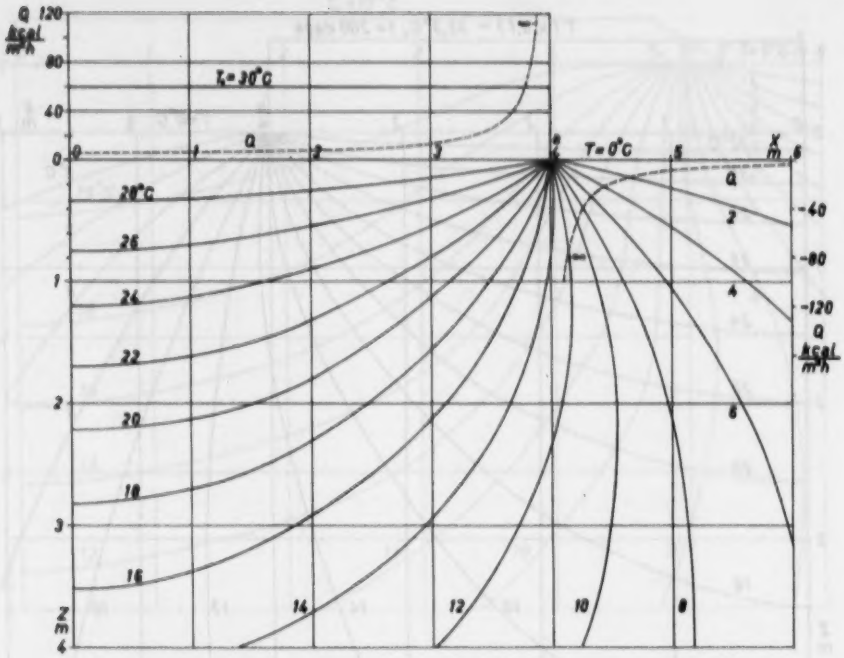

Fig. 21. Stationary temperature distribution field produced in the soil by a slab having the shape of a narrow strip, according to equation (30), and thermal flow from the slab into ground when $T_0 = 30\ ^{o}C$ and $\lambda_0 = 1$ kcal/m h ^{o}C.

the slab has gone through the variations shown in Fig. 13 during the preceding 200 days.

For $T(\tau) \equiv T_0$ and $t = \infty$, the appropriate limit process converts equation (28) into a less complex equation

$$T(x, z) = \frac{T_0}{\pi}\left[\text{arc tan}\ \frac{a + x}{z} + \text{arc tan}\ \frac{a - x}{z}\right] \qquad (30)$$

from which the corresponding stationary field is easy to calculate [12].

The stationary field calculated by equation (30) for $T_0 = 30\ ^{o}C$ and for the said slab having the shape of a narrow strip is shown in Fig. 21.

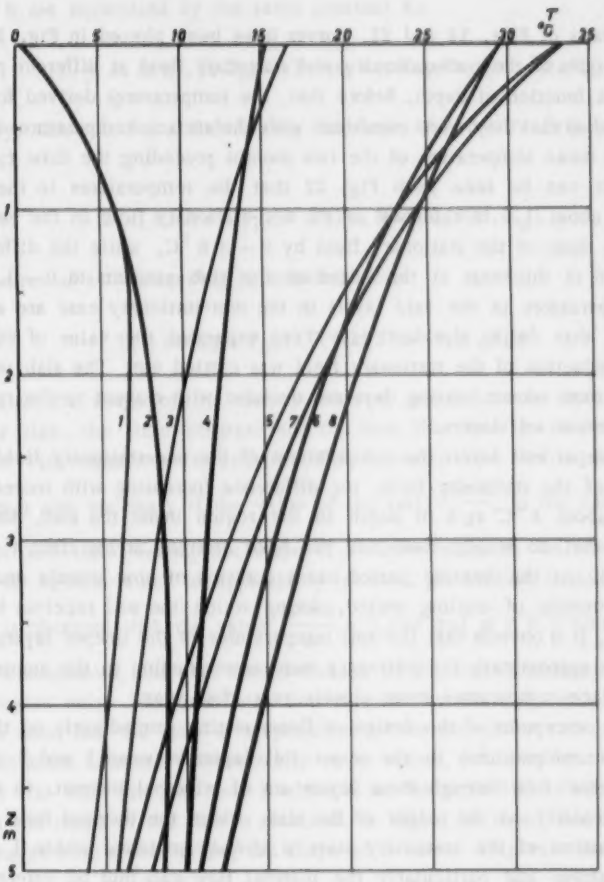

Fig. 22. Temperature distribution in the field produced in the soil by a slab having the shape of a narrow strip (a = 4,0 m) at different points under the slab:

Non-stationary field		Stationary field	
Curve No.	x =	Curve No.	x =
1	5/4 a	2	5/4 a
3	a	4	a
5	1/2 a	6	1/2 a
7	0	8	0

On the basis of Figs. 14 and 21, curves have been plotted in Fig. 22, showing the temperatures of the non-stationary and stationary field at different points under the slab as a function of depth. Before that, the temperatures derived from Fig. 21 were reduced so that they were consistent with the surface temperature of 29,5 °C, which is the mean temperature of the two months preceding the time t = 200 days (Fig. 13). It can be seen from Fig. 22 that the temperatures in the upper soil stratum of about 1,0 m thickness of the non-stationary field in the centre of the slab exceed those of the stationary field by 0 ... 3,8 °C, while the difference in a layer of 0,6 m thickness at the edges of the slab amounts to 0 ... 1,4 °C. The higher temperatures in the said layers in the non-stationary case are accountable to the fact that during the last days $T(\tau)$ exceeded the value of 29,5 °C, for which the reduction of the stationary field was carried out. The slab temperatures during the most recent heating days are decisive with respect to the temperatures in the uppermost soil layers.

In the deeper soil layers the temperatures of the non-stationary field are lower than those of the stationary field, the difference increasing with increasing depth and being about 4 °C at 5 m depth in the region under the slab, for instance. This shows that no steady state has yet been attained at the time t = 200 days (March 20th). As the heating period has a duration of nine months and there are only three months of cooling period, during which the soil receives heat in the summertime, it is obvious that the soil temperatures in the deeper layers under the slab tend to approximate the stationary state corresponding to the annual mean of the slab surface temperature more closely year after year.

From the viewpoint of the design of floors resting immediately on the ground, however, the temperatures in the upper soil layers between 1 and 2 m in depth and the thermal flow through these layers are of principal interest. In the surface layers, particularly on the edges of the slab, where the thermal flow is highest, an approximation of the stationary state is already achieved within 1 to 2 days. The temperatures and particularly the thermal flow can thus be determined with sufficient accuracy from the equation derived for the stationary state, by which the calculations are considerably facilitated.

The stationary temperature fields and the thermal flows from various kinds of slabs to the ground, as determined by the mathematical equations, have been shown in the figures for slabs of a given size and having a given surface temperature T_o. It is a simple matter to convert the results to correspondence with another surface temperature T, as this merely requires multiplication of the values read from the temperature field diagram by the ratio T/T_o. The heat flow, too, is directly proportional to the surface temperature and the conversion can be effected in the same manner.

When the size of the slab changes, its geometrical shape being retained, the temperature field under the new slab is obtained by altering the field representation in the desired ratio K. According to equation (4) the temperature in the field under a rectangular slab is not changed when the coordinates as well as the values

a, a' and b are multiplied by the same constant K:

$$T(x, y, z, a, a', b, T_o) = T(Kx, Ky, Kz, Ka, Ka', Kb, T_o)$$

Similarly, there is for a slab having the shape of a narrow strip, according to equation (6):

$$T(x, z, a, a', T_o) = T(Kx, Kz, Ka, Ka', T_o)$$

and for a circular slab according to equation (20):

$$T(r, z, R, \mu, T_o) = T(Kr, Kz, KR, \mu, T_o)$$

When also the shape of the slab is altered, which can be done only with a rectangular slab, the field diagram and the heat flow have to be recalculated in accordance with equations (4) and (5). This is easily done with a computer. Recalculation is also necessary if the value of the ratio $\frac{a' - a}{a}$, in the case of rectangular slabs and slabs having the shape of a narrow strip, or the coefficient μ in the case of circular slabs is appreciably altered. In normal buildings sufficient accuracy is obtained with the values $\frac{a' - a}{a} = 0,125$ and $\mu = 0,9$ employed in the preceding calculations, and also when the shape of the slab is changed in an insignificant degree only. The total heat flow supplied by the slab, and the heat flow from each zone into the soil, is proportional to the conversion ratio K, while the areas of the zones increase proportionally to its square, K^2. The heat flow per unit area from the slab into the soil is changed in the ratio 1:K at any point of the slab when the shape of the slab is retained. It should be noted that the heat flow per unit length from a slab having the shape of a narrow strip into the soil remains constant when the ratio $\frac{a' - a}{a}$ remains unchanged even if the width of the strip varies.

If no computer is available for the calculations, the following approximate method can be used. A known rectangular slab is altered in the manner described in the foregoing so that its width will equal that of the slab under consideration. The isotherms at the centre of the cross sections of the slabs are thus in good mutual agreement. However, the temperatures in the isotherm field of the slab under consideration are somewhat lower or higher than those in the known field, depending on whether the slab under consideration is shorter or longer than the slab derived by conversion from the known slab.

On comparing the temperature field under a rectangular slab in the centre of its cross section (Fig. 6, p. 22) and that of a slab having the shape of a narrow strip, with the same width and the same surface temperature, $T_o = 30\ ^oC$ (Fig. 10, p. 25), the observation can be made that the fields are mutually rather little different in this

case too. For instance, the temperatures in the first-mentioned field at 1,5 m depth under the centre of the slab are 1 °C lower and those at the edge in the middle of the longer side are 0,8 °C lower than those for the latter. At greater depth there are greater differences; e.g., at 3 m depth they amount to 1,9 and 1,8 °C, respectively. The field representation for a slab having the shape of a narrow strip, which can be obtained by means of the simple equation (6), can already be considered a relatively good approximation also of the temperature field under a rectangular slab, in a cross section through the central region of the slab.

The temperatures in the cross sections of two rectangular slabs having the same width differ less than those in the fields of a rectangular slab and one having the shape of a narrow strip.

The temperature fields show conformity with a good degree of accuracy also at the ends of the slab.

In the approximate calculation of the total thermal flow from rectangular slabs of different shapes and sizes, the slab with known thermal flow is converted to have the same width as the slab under consideration. This is done by multiplying the dimensions of the known slab by the ratio of the widths, and its thermal flow will then also have to be multiplied by this ratio. The resulting slab may have greater or smaller length than the slab under consideration. Its thermal flow has then to be decreased or augmented by the thermal flow of a slab having the shape of a narrow strip, which has the same width and a length equalling the difference in length between the two slabs. The thermal flow from the known rectangular slab and the said slab having the shape of a narrow strip can be taken from Tables 4 and 5 on pp. 86 and 88.

2. EXPERIMENTAL HOUSES AND MEASUREMENTS

21. LOCATION, STRUCTURAL DESIGN AND SOIL TYPE

In this investigation, use was made of one-family houses at Tapiola, 10 km distant from Helsinki, as experimental houses. The houses were erected immediately on the ground and had a floor area of about 100 m^2 and a heated space of 300 m^3 capacity. They were low, bungalow-type houses built of light-weight concrete (Fig. 23).

The houses were erected on a concrete slab supported directly by the soil surface and heated with steel tube elements embedded in the concrete. The thermal energy was supplied by the remote heating plant of the area.

Measurements were further carried out in the building of the secondary school of Tapiola. This building, too, has a concrete floor laid immediately on the ground, but it is heated by means of radiators.

Altogether 26 one-family houses of the type described above have been erected on sites with different kinds of soil. Three of them were chosen to serve as experimental houses in this investigation and one of the three, Experimental House No. 1, was subjected to highly detailed study. All experimental houses have identical floor slabs and identical heating tubes within the slab, except that the thermal lagging layer is different in the different houses. Experimental House No. 1 and 2 have been built on a moraine hillock, the thickness of the soil layer between the floor slab and bedrock varying between 0,6 and 2 m. Experimental House No. 3 stands on a formerly cultivated field with clay base soil. The secondary school of Tapiola (Experimental House No. 4) is located on heath land.

Structure of the floor slab. The structure of the floor slab and the position of the heating tubes in Experimental House No. 1 have been shown in Figs. 24 and 25. The heating elements and distributing pipes consist of steel tubing. The heating elements contain the following total lengths of various pipe

Fig. 23. Experimental House No. 1 in wintertime conditions. The sides of the house have thick snow walls; on the gavel ends the snow has the same thickness as upon free ground.

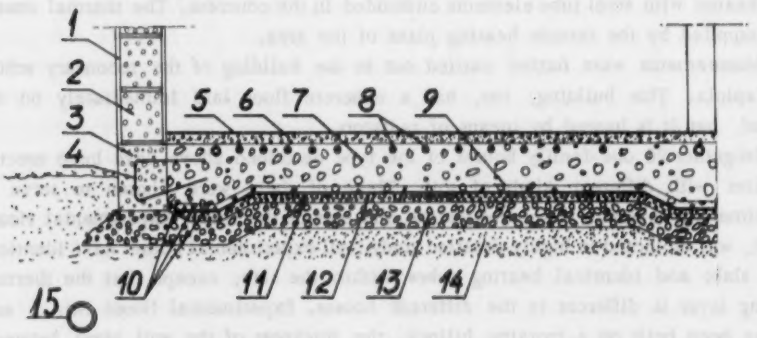

Fig. 24. Floor slab structure in Experimental House No. 1. - 1 — 2: Light-weight concrete, 3: Light-weight gravel concrete, 4: Iron clamps, 5: Linoleum flooring, 6: Levelling concrete, 7: Foundation slab, 8: Heating pipes, 9: Reinforcement netting, 10: Steel reinforcements, 11: Bitumen roofing paper, 12: Light-weight gravel course, 13: Shingle course, 14: Earth filling, 15: Drain.

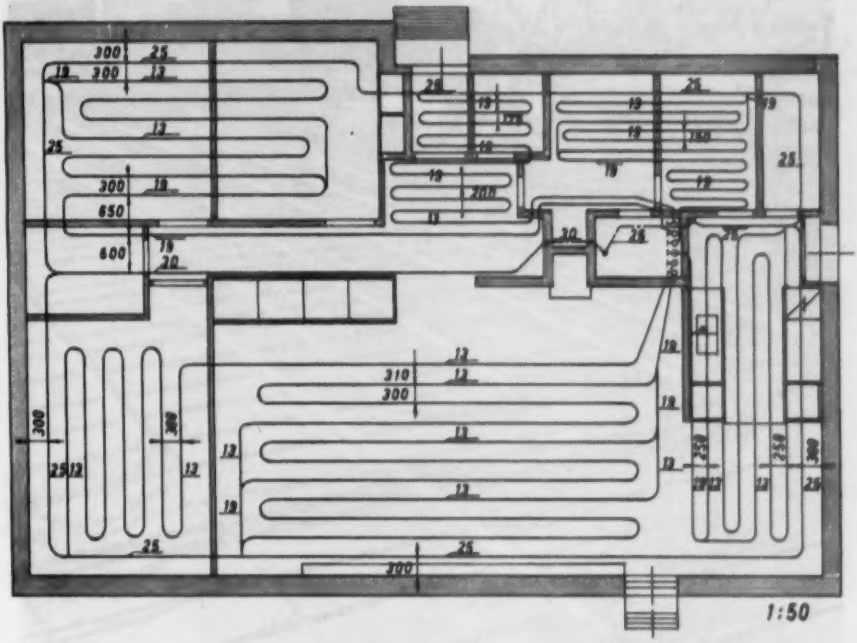

Fig. 25. Location of the heating pipes, pipe sizes, and pipe spacing (in mm).

sizes: 13 mm pipe, 196 m; 19 mm, 44 m; 25 mm, 45 m; and 32 mm, 10 m. The pipe spacing, different in the different elements, can be seen in Fig. 25. The arrangement of the heating elements represents an attempt to achieve uniform heating of the slab under the different rooms. Since the heat losses are greater on the edges than in the centre of the slab, the flow of the water constituting the heating medium is directed so that the water temperature decreases in the course of flow from the edges of the slab towards its centre.

The following procedure was applied in the erection of Experimental House No. 1. Prior to pouring of the concrete for the foundation slab, the top layer of the soil with organic constituents was peeled off and the soil surface was levelled. The levelled moraine was overlaid with a shingle layer of 10 cm thickness, intended to prevent the capillary ascent of water from the soil into the foundation slab and also to serve as thermal lagging. Additional thermal insulation was provided by a light-weight gravel layer of 4 cm, covered in its turn with thick bitumen paper (weighing 2,25 kg per m^2) with 30 cm overlapping of the strips. The purpose of this layer was to prevent the concrete mix from penetrating the light-weight

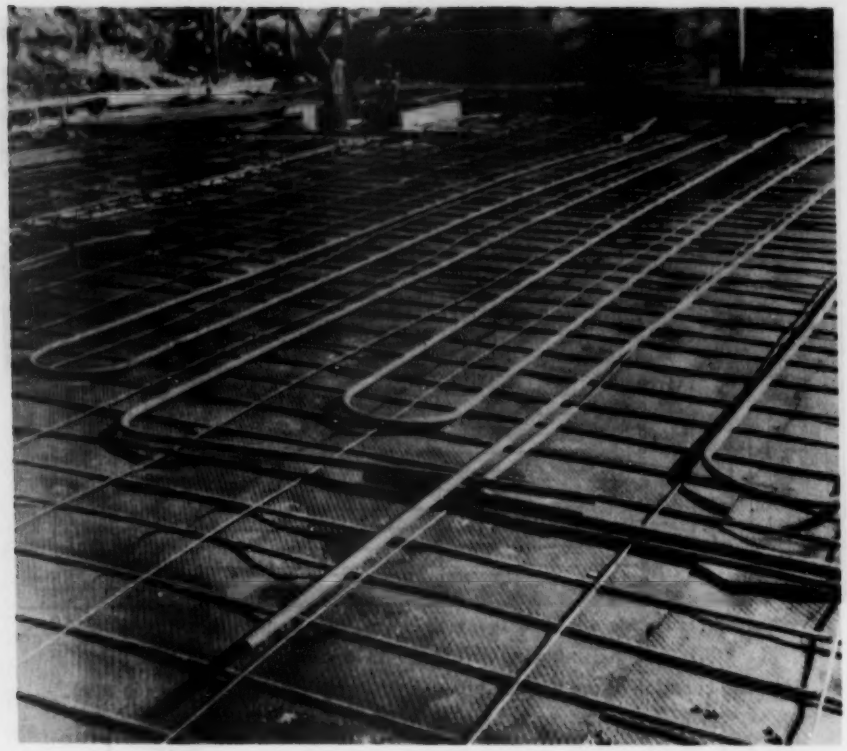

Fig. 26. Heating pipes, steel reinforcements and roofing paper after their installation and prior to concreting of the floor.

gravel and shingle layers and to serve continuously as a vapour barrier, preventing the access of the moisture occurring in the form of vapour in the soil into the floo structure. The circumference of the building was laid of 25 cm x 50 cm x 30 cn light-weight gravel-concrete cavity blocks and the concrete to make up the slal was poured into the through that was formed in this manner. Prior to pouring, the steel reinforcements, heating pipes and thermocouples for measuring purposes were inserted in the mould (Fig. 26). Ultimately, the floor was levelled with an additional concrete layer about 5 cm in thickness and this was covered with 2,5 mm linoleum cemented to the concrete.

The other two one-family houses selected to serve as experimental houses have the same floor structure, except that a shingle layer of 15 cm is the sole thermal lagging in Experimental House No. 2, while Experimental House No. 3 has a

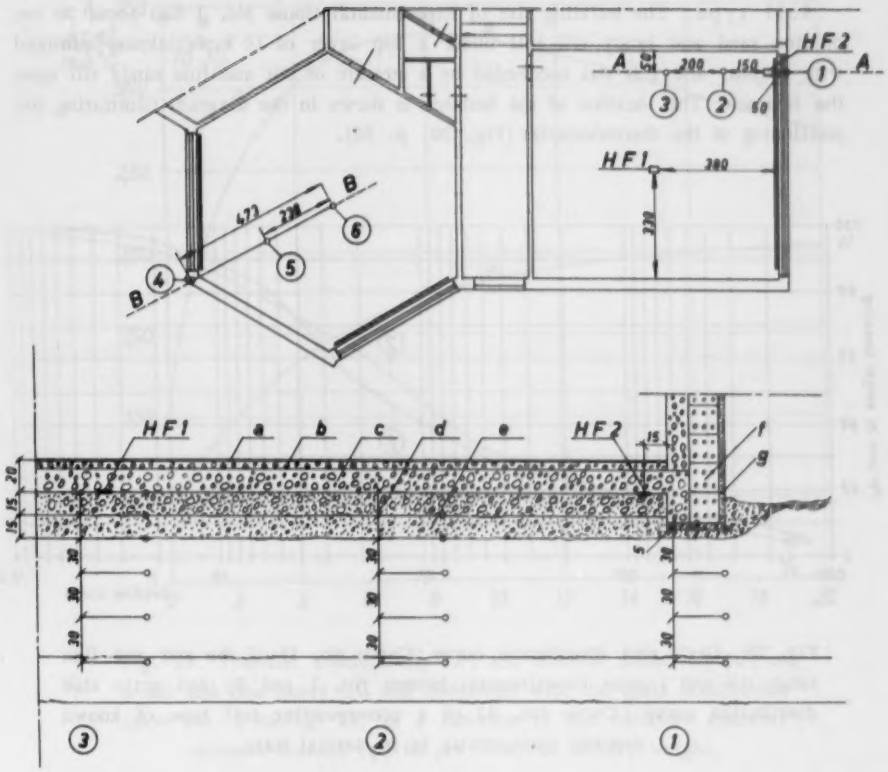

Fig. 27. Structural design of the unheated floor laid immediately on the ground (Experimental House No. 4), location of thermocouples and heat quantity meters. - a: Linoleum flooring, b: Levelling concrete, c: Foundation slab, d: Light-weight gravel course, e: Earth filling, f: Light-weight gravel concrete, g: Plaster covering, 1 ... 6: Thermocouple groups, HF_1, HF_2: Heat flow meters.

light-weight gravel layer of 10 cm and a shingle layer of 20 cm thickness. Furthermore, the design of the slab in Experimental House No. 3 is different in that it has a foundation consisting of piles, owing to the soft soil, and there are concrete beams 50 cm in thickness and 20 cm in width, embedded in the soil, on its edges and in the centre. The school building of Tapiola (Experimental House No. 4) has an unheated floor slab of the shape and design shown in fig. 27.

Soil type. The building site of Experimental House No. 1 had about 30 cm of fine sand and sandy till soil under a top layer of 10 cm thickness admixed with humus, and this was succeeded by a stratum of silt and fine sandy till upon the bedrock. The location of the bedrock is shown in the diagram illustrating the positioning of the thermocouples (Fig. 30, p. 53).

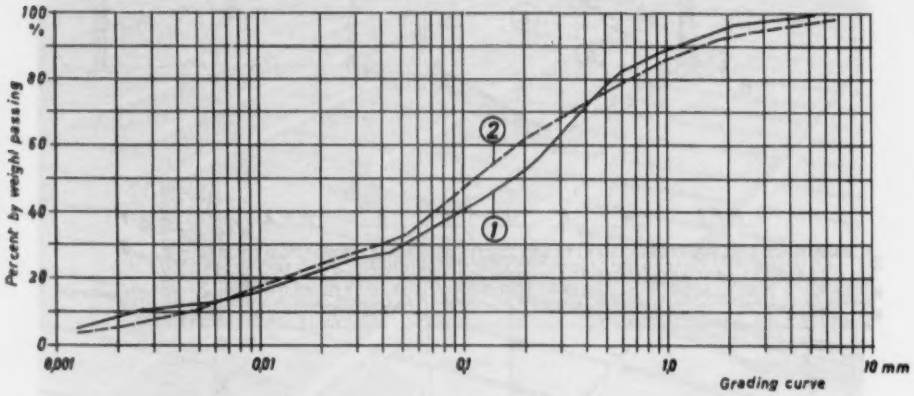

Fig. 28. Grain size distribution curve (Curve No. 1) of the silt and fine sandy till soil (under Experimental Houses No. 1 and 2) and grain size distribution curve (Curve No. 2) of a corresponding soil type of known thermal conductivity in its natural state.

For determination of the volume weight of the soil, a cylindrical pit was dug in the ground and the soil removed from the pit was weighed. The volume removed from the pit was determined by filling the pit with water after it had been sealed with a plastic film. The soil itself as well as the earth filling, consisting of till soil taken from the building site, contained stones in considerable quantity and the best attempts were made to include stones in the samples in correct proportion. The dry volume weight of the fine sand and sandy till forming the top layer, in its natural state, was found to be 1629 kg/m^3 and that of the silt and fine sandy till base soil, 1862 kg/m^3.

The grain size distribution curve of the silt and fine sandy till layer, shown in Fig. 28 (Curve No. 1), was determined by The Geological Survey of Finland. The thermal conductivity of a soil presenting approximately the same grain distribution (Fig. 28, Curve No. 2), in terms of the moisture content, is shown

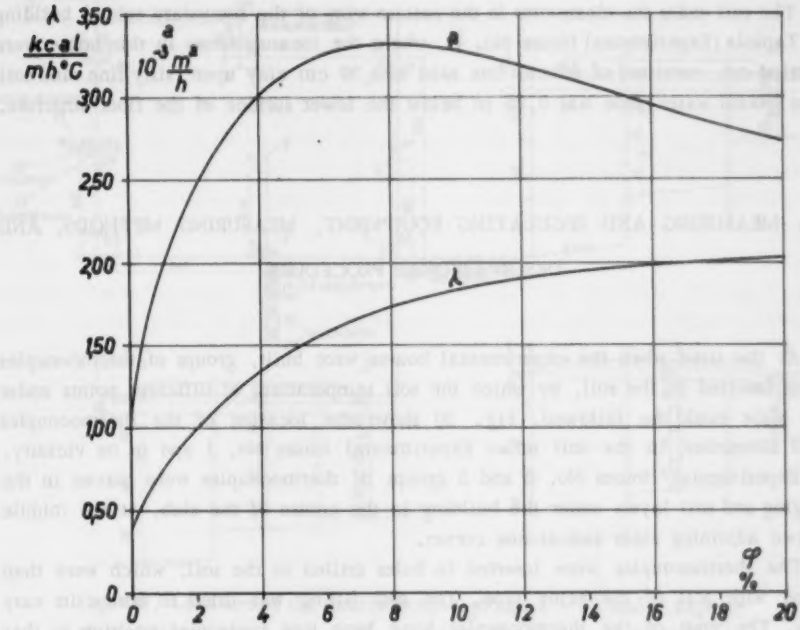

Fig. 29. Thermal conductivity (λ) and thermal diffusivity (a) of a silt and fine sandy till soil consistent with the grain distribution curve No. 2 in Fig. 28, as a function of the moisture content. Moisture content (φ) in per cent. of the dry weight.

in Fig. 29. This curve has been plotted in accordance with the investigations of Saare [17]. The thermal diffusivity, calculated as a function of moisture content, has also been plotted in this figure.

Since the heat-conducting properties of the fine sand and sandy till layer are quite close to the characteristics of the silt and fine sandy till layer, the values applying to the latter, thicker layer are employed for the entire soil layer (Fig. 29).

The soil under Experimental House No. 2, adjacent to Experimental House No. 1, is of the same kind as that under the latter.

Topmost in the soil at the site of Experimental House No. 3 was a humus layer, which was removed, succeeded by silt in a thickness of 25 cm, under which there was heavy clay, about 50 cm, and finally muddy clay down to the bedrock, which was found at a depth of 4 – 5 m.

The soil under the classrooms in the eastern wing of the secondary school building of Tapiola (Experimental House No. 4), where the measurements in this house were carried out, consisted of 80 cm fine sand and 30 cm clay upon silty fine sand soil. The ground water table was 0,85 m below the lower surface of the floor structure.

22. MEASURING AND REGULATING EQUIPMENT, MEASURING METHODS, AND OBSERVATIONAL PROCEDURE

At the time when the experimental houses were built, groups of thermocouples were inserted in the soil, by which the soil temperatures at different points under the slabs could be followed. Fig. 30 shows the location of the thermocouples (119 altogether) in the soil under Experimental House No. 1 and in its vicinity. In Experimental Houses No. 2 and 3 groups of thermocouples were placed in the lagging and soil layers under the building in the centre of the slab, at the middle of two adjoining sides and at one corner.

The thermocouples were inserted in holes drilled in the soil, which were then filled with soil of the same type. The soil filling was dried to ensure its easy flow. The ends of the thermocouples were bent into horizontal position so that the leads ran horizontally for a few centimetres at the same depth at which the thermocouple junction was placed; this was done in order to minimize the errors due to heat conduction.

For the purpose of determining the mean temperature at the level of the heating tubes, thermocouples were placed in the slab between the pipes in the fields with different spacing of the pipes, both at the height of their centre lines and at corresponding points on the lower surface of the concrete slab and on its upper surface, under the levelling concrete layer (Fig. 31). These thermocouples were used to determine the mean temperature at the pipe level, T_o, and the mean temperatures in the measuring planes below and above this level, T_1 and T_2, respectively. These data were required in order to determine the distribution of the supplied heating energy Q_c into downward and upward heat flow.

When the distances d_1 and d_2 of the thermocouple planes (T_1 and T_2, Fig. 31) from the pipe level (T_o) are known and the mean temperatures at all three levels have been determined by measurement, the ratio of the downward and upward heat flow is found from the equation

$$\frac{Q_1}{Q_2} = \frac{k_1 (T_o - T_1)}{k_2 (T_o - T_2)} \qquad (31)$$

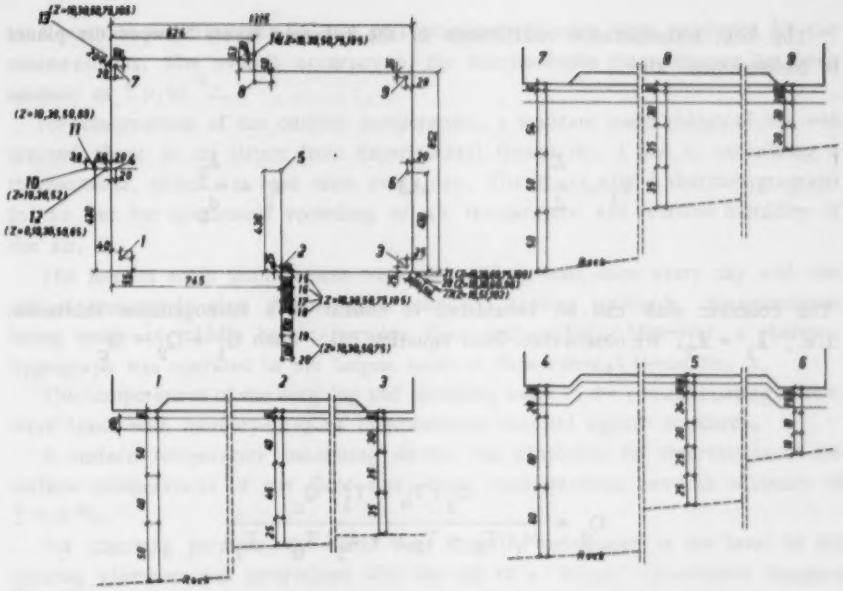

Fig. 30. Location of the thermocouple groups 1 ... 23 under Experimental House No. 1 and in its vicinity. The depths of the individual thermocouples in groups 10 ... 23 are indicated by the z(cm) values entered in the figure.

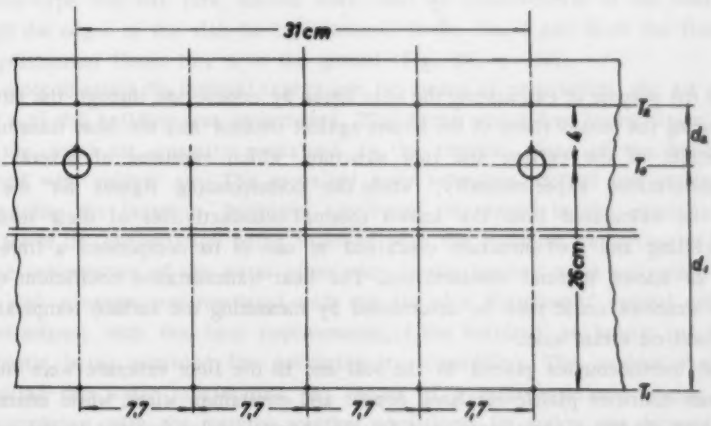

Fig. 31. Location of the thermocouples in the plane of the heating pipes (temperature T_0) and in the measuring planes below and above (T_1 and T_2).

The heat transmittance coefficients of the concrete layers between the planes of measurement are:

$$k_1 = \frac{\lambda_1}{d_1} \qquad \text{and} \qquad k_2 = \frac{\lambda_2}{d_2}$$

The concrete slab can be considered to consist of a homogeneous substance, i.e., $\lambda_1 = \lambda_2$. We obtain then from equation (31) when $Q_1 + Q_2 = Q_c$:

$$Q_1 = \frac{d_2 (T_o - T_1)\, Q_c}{d_1 (T_o - T_2) + d_2 (T_o - T_1)} \qquad (32)$$

$$Q_2 = \frac{d_1 (T_o - T_2)\, Q_c}{d_1 (T_o - T_2) + d_2 (T_o - T_1)}$$

For the purpose of calculating the heat losses by conduction through the structures delimiting the heated space of the houses against outdoor air, the heat transmittance coefficient of the ceiling and roof structure, which consisted of several layers, was determined experimentally, while the corresponding figures for the walls could be calculated from the known thermal conductivities of their materials. The ceiling and roof structure contained as one of its components a fibre board sheet of known thermal conductivity. The heat transmittance coefficient of the entire structure could then be determined by measuring the surface temperature of this sheet on either side.

The thermocouples placed in the soil and in the floor structure were made of 0,5 mm diameter plastic-insulated copper and constantan wires; while enamelled, 0,25 mm diameter wires were used for the thermocouples inserted in the ceiling. Consisting of material from one and the same manufacturing batch, all 0,5 mm wires were of uniform quality. All thermocouples were checked at 0 °C and part of them were calibrated in the temperature range from 0 to 35 °C.

A portable potentiometer and two potentiometer recorders were employed for the measurements. The overall accuracy of the thermocouple measurements has been assessed as ± 0,25 °C.

For observations of the outdoor temperature, a standard meteorological hut was erected about 30 m distant from Experimental Houses No. 1 and 2, containing a thermometer, which was read once every day. There was also a thermohygrograph in the hut for continuous recording of air temperature and relative humidity of the air.

The interior room temperatures were observed at least once every day with the aid of mercury-in-glass thermometers checked against standards, measurements being made at middle height between floor and ceiling. Moreover, a thermohygrograph was operated in the largest room of Experimental House No. 1.

The temperatures of the outgoing and returning water in the central heating system were taken with mercury-in-glass thermometers checked against standards.

A surface temperature measuring device was employed for observations of the surface temperatures of the floor slab; these measurements have an accuracy of ± 0,5 °C.

For checking purposes, the total heat quantity introduced at the level of the heating elements was determined with the aid of a "Pollux" calorimeter designed to special order. The calorimeter was normally read once every day, but more frequent readings were taken for occasional determinations of the momentary heat consumption. The errors inherent in this measuring device were taken into account on the basis of a correction graph that had been plotted for the instrument. After this correction, the measurements of the heat quantities with the calorimeter have an accuracy of ± 2 %.

Sheet-type thermal flow meters were used for measurement of the heat losses through the edges of the slab in Experimental House No. 1 and from the floor slab of Experimental House No. 4 to the ground (Fig. 27, p. 49).

In order to assess the thermal energy lost by means of ventilation, the air change factor n of the building was determined. This factor states how many times in one hour the total air quantity contained in the interior space of the building is replaced with outdoor air. The so-called trace substance method was employed in determining the factor n, hydrogen gas being introduced in the rooms and the decrease of its concentration being followed with the hydrogen catharometer [19].

The temperature of the water circulating in the heating tubes embedded in the floor slab structure was regulated with the aid of a "Satchwell" central controller in accordance with the heat requirements of the building, an indoor and outdoor thermostat being provided for governing the controller. The outdoor thermostat controlled the water temperature, by means of a motor-operated mixing valve, in accordance with the varying weather conditions; its action was determined by the combined effect of prevailing outdoor temperature, wind, rain and solar radiation. The indoor thermostat was arranged to shut off the heating if excess heat was generated electrically, by a fire in the open hearth, etc. This regulating

equipment maintained nearly constant temperature, 20 ± 0,5 °C, throughout the period during which the house was heated. In springtime, when the heating effect of the sun's radiation exceeds the heat requirements of the house in the middle of the day, the indoor temperature will exceed the stated value during a few hours although the supply of hot water to the heating elements is entirely suspended through the controller action.

23. RESULTS OF MEASUREMENT

231. Temperatures of the slab and of the underlying soil

As a consequence of varying weather conditions there are continuous variations in the rate of thermal energy introduced in the slab and in the temperatures of the floor slab structure itself as well as the underlying soil. Transient variations are rapidly attenuated with increasing depth in the soil, and the soil temperatures at greater depths are determined by the long-term variations, in the first place by the fundamental wave having a period equal to one whole year.

The variation of the mean temperature at the heating tube level, $T_o(t)$, in Experimental House No. 1 during the heating period of 1956 – 1957 can be seen in Fig. 32. Moreover, this figure contains curves representing the variation of the temperature in the soil under different points of the slab and the corresponding temperature curves for free soil.

In the soil layers close to the slab, the soil temperature increases with increasing heat requirements of the house, as does also the temperature at the heating tube level; this increase continues until about January 10th. The sudden temperature rise in the soil layers under the house between the 7th and 10th of January was caused by snowfall during the time January 5th to 10th. Quantities of snow that had fallen on the roof, and thence on the ground, formed an efficient insulating layer on the edges of the slab, increasing the temperatures under the slab by 3 °C on an average during the said time.

The temperature field that existed under Experimental Houses No. 1 and 2 and in the intervening terrain at the end of the coldest period of the year, when both houses had approximately the same temperature in their heating tube plane, i.e., 33 °C, is shown in Figs. 33 and 34, obtained by measurements on the 20th of March, 1957. In the margin of these figures also the temperatures in undisturbed soil about 30 m distant from the experimental houses have been entered. Moreover, Fig. 35 shows the temperature field under Experimental House No. 1 on March 20th, 1957, in greater detail, revealing also the course of the isothermal lines in the floor slab structure and the influence of the heating elements upon them. It is evident from this diagram that the concrete slab heats the soil considerably in comparison with the surrounding area. For instance, the temperature at 10 cm depth under the centre of Experimental House No. 1 was 26 °C higher on the 20th

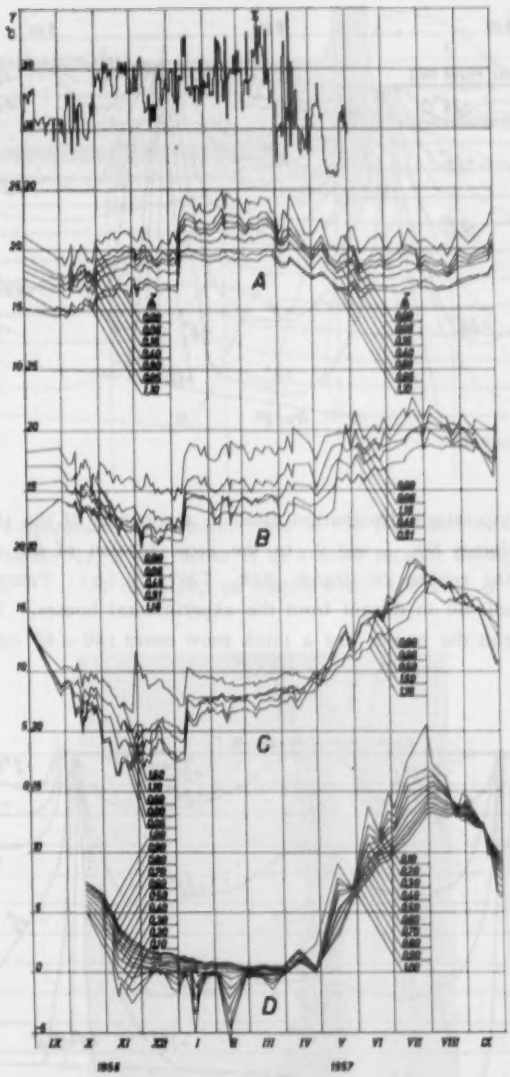

Fig. 32. Experimental House No. 1.- Variation of the mean temperature at heating pipe level, T_o, and temperature variations under the house during the time September 1st 1956 to September 30th, 1957, at the centre of the slab (A), in the middle of the long side (B) and at the corner of the slab (C), and variation of the temperatures in undisturbed soil 30 m distant from house (D). The depths z in groups, A, B and C refer to the lower surface of the concrete slab; in group D, to the soil surface (cf. Fig. 30).

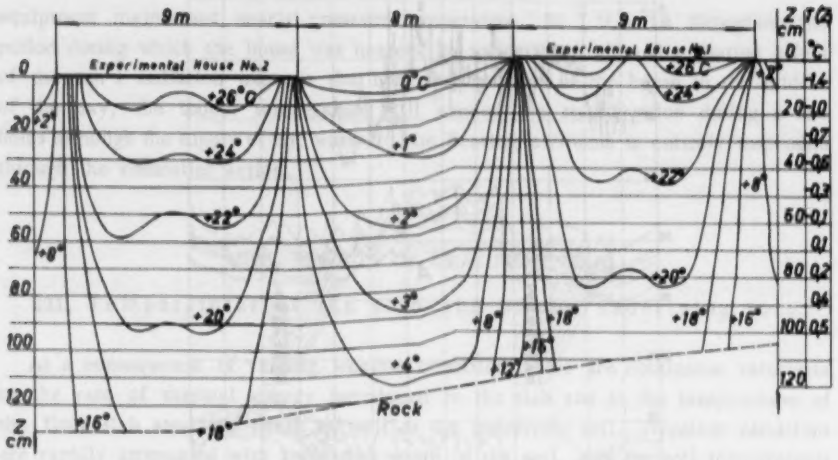

Fig. 33. Temperature distribution field at the centre of the slabs of Experimental Houses No. 1 and 2, in the cross section of the slab and in the intervening terrain on March 20th, 1957. T (z): Temperatures in undisturbed soil 30 m distant from the experimental houses.- The terrain between the houses had a thick snow cover (40 — 60 cm).

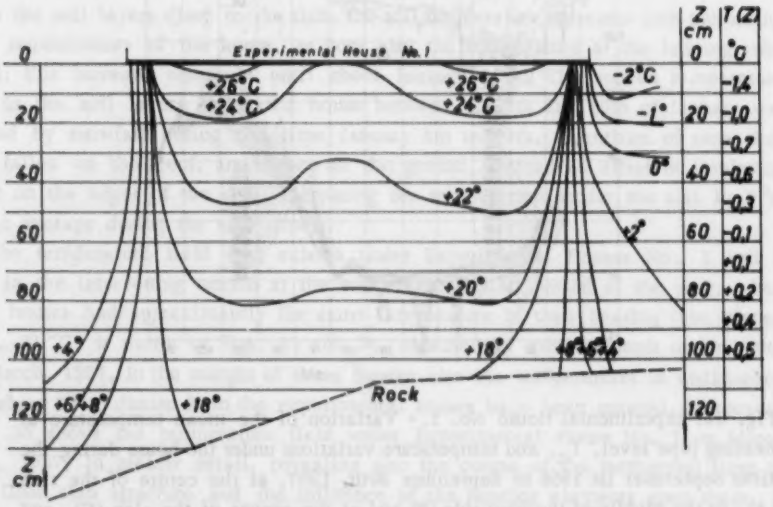

Fig. 34. Temperature distribution field in a section through the diagonal of the slab of Experimental House No. 1 on March 20th, 1957. T(z): Temperatures in undisturbed soil 30 m distant from the experimental house.

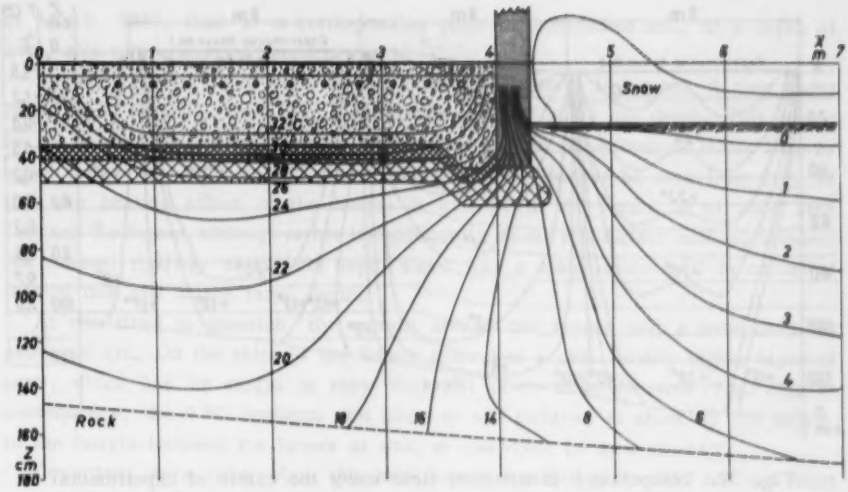

Fig. 35. Temperature distribution field in the floor slab structure of Experimental House No. 1 and under it in a cross section through the centre of the slab, on March 20th, 1957.

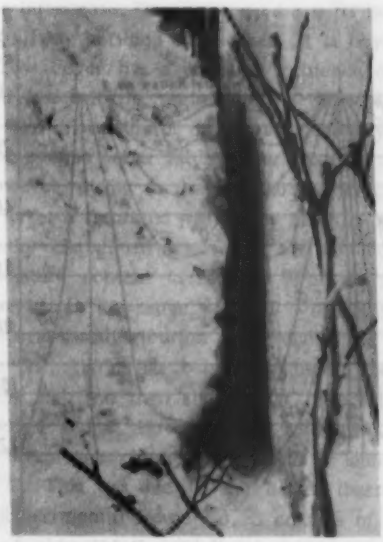

Fig. 36. Showing that an air gap has come into existence by melting between the snow wall and the house.

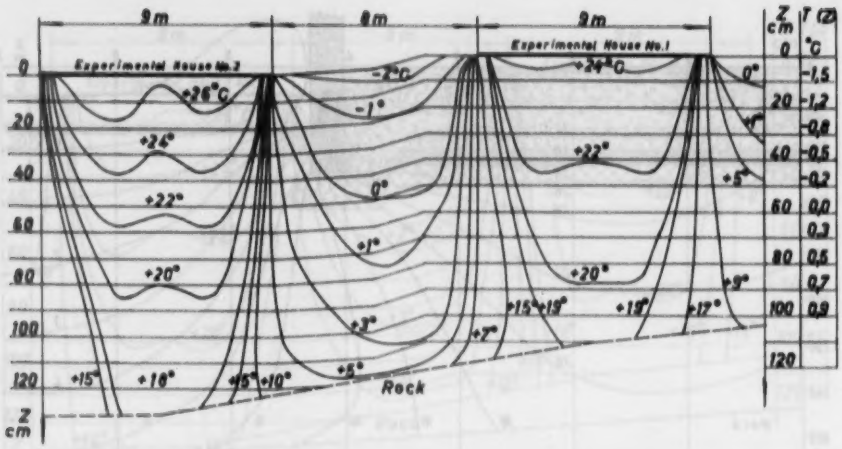

Fig. 37. Temperature distribution field under the centre of Experimental Houses No. 1 and 2 in the cross section of the slab, and in the intervening terrain, on January 18th, 1957.- The intervening terrain was covered by a thin, trampled snow layer. T(z): Temperatures 30 m distant from the experimental houses.

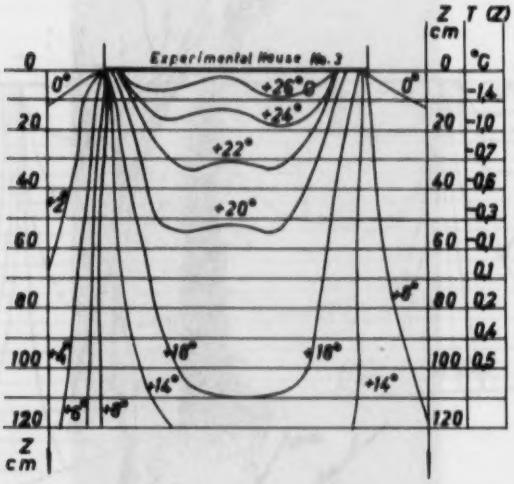

Fig. 38. Temperature distribution field under the centre of Experimental House No. 3 in the cross section of the slab on March 20th, 1957. T(z): Temperatures in undisturbed soil 25 m distant from the experimental house.

of March, 1957, than at a corresponding point in undisturbed soil; at a depth of 1 m, this temperature difference was 19 °C.

The soil temperatures were higher under Experimental House No. 2 than under No. 1; for instance, there was a difference of 3 °C at 10 cm depth. This is due to the poorer thermal insulation of the floor slab in Experimental House No. 2, which lacks the light-weight gravel course. It can further be seen from Fig. 33 that the heating effect of the houses is evident in the entire, 8 m wide area between the houses, although rather insignificantly on the soil surface halfway between the houses. Halfway at 120 cm depth there was a temperature 3 °C in excess of that of free soil at the same depth.

At the time in question, the terrain around the houses had a snow cover of about 50 cm. On the sides of the houses there was a considerably higher layer of snow, which had its origin in snow that fell down from the roof (Fig. 36). In consequence, the 0 °C isotherm runs close to soil surface, at about 10 cm depth, in the terrain between the houses as well as elsewhere in their vicinity.

Conditions at a time when the snow cover was still quite thin and had been partly removed from the space between the houses, are illustrated by Fig. 37, showing the temperature field found in measurements that were carried out on January 18th, 1957. The temperature fields under Experimental House No. 1 and No. 2 were approximately equivalent at both times represented by Fig. 33 and Fig. 37, respectively, but the temperatures in the area between the houses were considerably lower in the latter case. For instance, the 0 °C isotherm was then found at about 0,5 m depth halfway between the houses. It is obvious on comparison of Figs. 33 and 37 that snow cover has a profound influence on the temperatures in the soil and on the heat losses in the regions at the edge of the slab.

Experimental House No. 3 was built on a wet, formerly cultivated field with clay undersoil, where the ground water table came close to the soil surface, i.e., about 0,85 m down. However, owing to the capillary water conduction in the clay stratum, the entire clay layer is saturated with water and the ground water line does not imply any noteworthy change in the soil's thermal conductivity. The clay layer, which had a moisture content of 50 ... 80 % of its total weight, had low thermal conductivity varying in the range λ = 1,0 ... 0,7 kcal/m h °C.

In Fig. 38, the temperature distribution field under the slab of Experimental House No. 3 on March 3rd, 1957, has been reproduced. The temperature in the heating tube plane of the slab was then about 35 °C, i.e., slightly higher than in Experimental Houses No. 1 and 2. However, the soil surface temperatures under the floor slab structure were approximately the same as under Experimental House No. 1, about 26 ... 27 °C. This was due to the better thermal lagging under the slab. This lagging in Experimental House No. 3 consists of a 10 cm light-weight gravel layer and it is further improved by the air gap that has come into being here and there between the slab and the light-weight gravel, owing to shrinkage of the clay soil. The temperature falls at a considerably greater rate with increasing depth than under Experimental House No. 1. This is attributable to the

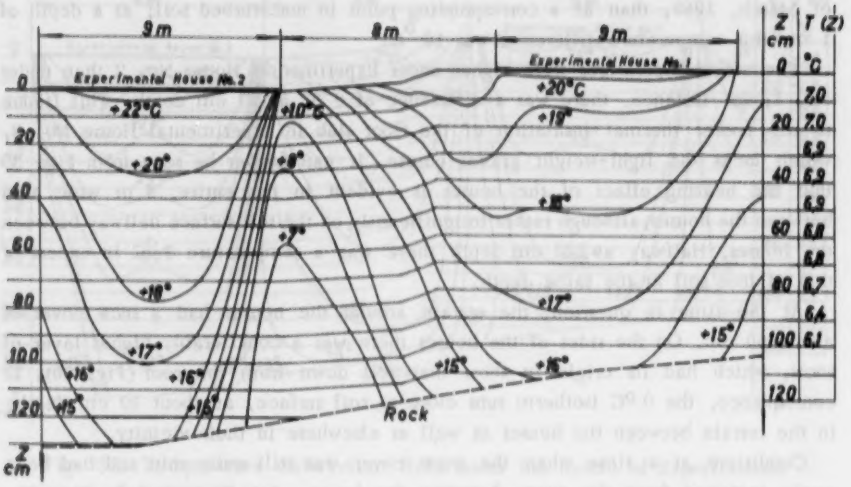

Fig. 39. Temperature distribution field under the centre of Experimental Houses No. 1 and 2, and in the intervening terrain, at noon on May 28th, after conclusion of the heating period. T (z): Temperatures in undisturbed soil 30 m distant from the experimental houses.

lower thermal conductivity of the top layers.

From the temperature distributions and other results of measurements described in the foregoing the inference can be drawn that penetration of ground frost into the soil under the experimental houses at their sides is not possible, seeing that the 0 °C isotherm runs sufficiently far outside the house even in the case that the ground has been swept clean of snow (cf. Fig. 37). On the other hand there is a greater risk of ground frost on the corners, as can be seen from Fig. 34. At the corner marked as No. 3 (Fig. 30), the 0 °C isothermal line actually runs in the insulation layer of the slab. This is mainly due to the fact that the snow was trampled tight in the space around this corner. Cooling of the soil at the corners is furthermore enhanced by the circumstance that the water flowing down through the down pipes wets the soil in the autumn, increasing its thermal conductivity. Therefore, the rain water should be diverted to points farther away from the corners, and the corners should be provided with efficient external thermal lagging and insulation against moisture.

It is also interesting to study the isotherms in the soil in summertime from the viewpoint of the soil as a factor influencing the surface temperature of the floor slab when the heating system is inoperative. Fig. 39 shows the course of the

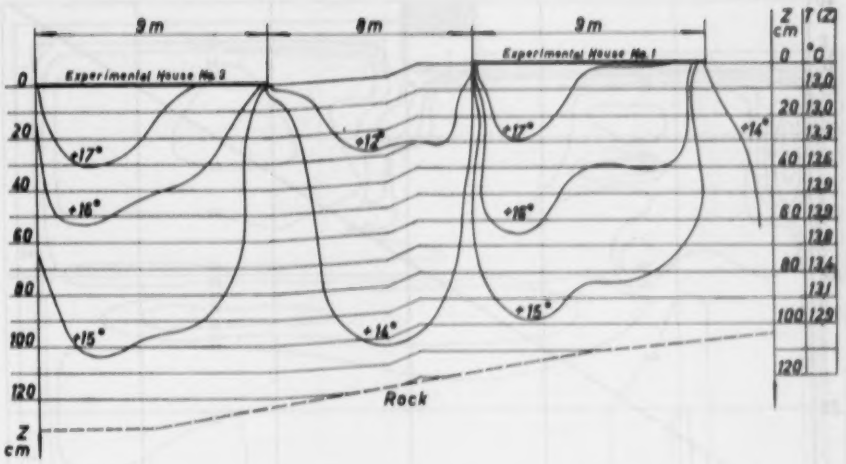

Fig. 40. Temperature distribution field under the centre of Experimental Houses No. 1 and 2 in the cross section of the slab, and in the intervening terrain, on August 24th, before commencement of the heating period.
T(z): Temperatures 30 m distant from the experimental houses.

isothermal lines in the spring, that is, at noon on May 29th, 1957. It is seen that the sun heats the soil on the south side of the house and somewhat also on its east and west gavel ends. Consequently, also the floor temperatures on these edges are higher than at the north edge of the slab. In cold summers, as in the summer of 1957, the surface temperature of the slab is too low, only about 17 °C, particularly in the rooms lying towards the north. In hot summers, e.g. in the summer of 1959, the slab has been found to exert a desirable cooling effect on the room temperatures.

Fig. 40 shows the course of the isothermal lines under Experimental Houses No. 1 and 2 and in the intervening terrain on August 24th, 1957, before commencement of the heating period. The soil temperature has already begun to go down in the intermediate terrain, but the temperatures under the houses are still the same as in midsummer.

Accurate determination of the mean temperature at the heating pipe level in the floor slab is possible only by means of special arrangements, e.g., with the aid of thermocouples embedded in the slab. However, its approximate value can be found in actual practice from the mean of the temperatures at which the circulating water enters the heating pipe system and which it has on its return.

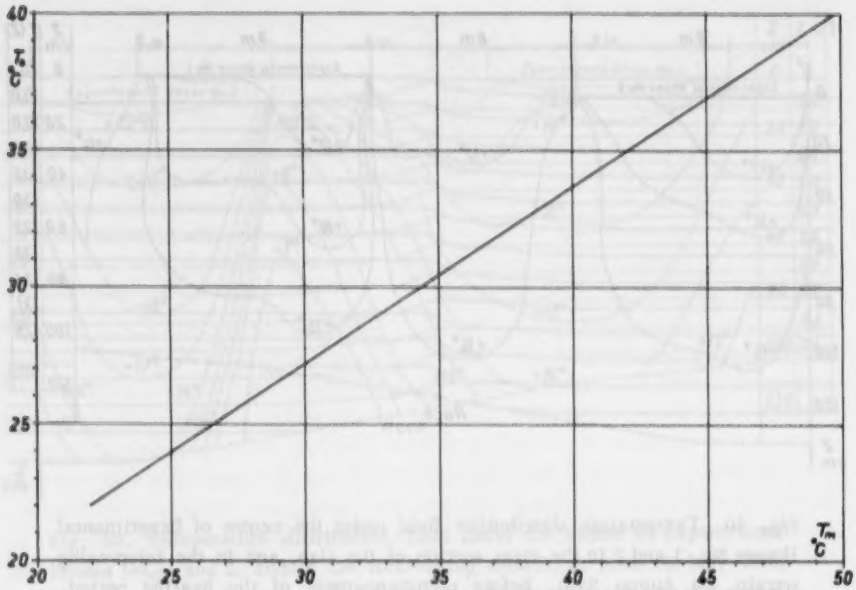

Fig. 41. Dependence of the mean temperature T_o at heating pipe level in Experimental House No. 1 of the mean of the outgoing and incoming heating water temperatures, T_m.

These temperatures can be ascertained with the aid of thermometers mounted in the pipe system in the normal manner. Fig. 41 shows the relationship between the mean temperature at the heating pipe level in the floor slab and the mean of outgoing and incoming water temperatures, as observed in Experimental House No. 1. This figure provides data that can be useful if the layout of the heating pipe system is not greatly different from that in Experimental House No. 1 (Fig. 25, p. 47). With closer spacing of the pipes, the mean temperature at pipe level will approach the mean of the outgoing and incoming water temperatures.

The temperature distribution on the upper surface of the floor slab in Experimental House No. 1, measured without any carpets on the floor, can be seen in Fig. 42 for a condition when the mean of the outgoing and incoming water temperatures was 34,5 °C and the outdoor air temperature was -11,2 °C. According to Chrenko[20], the temperature of the floor surface may not exceed 25 °C even in coldest weather without sacrifice in dwelling comfort. Other investigators, e.g. Newis and Flinner[21], permit a maximum of 35 °C. In our Experimental House

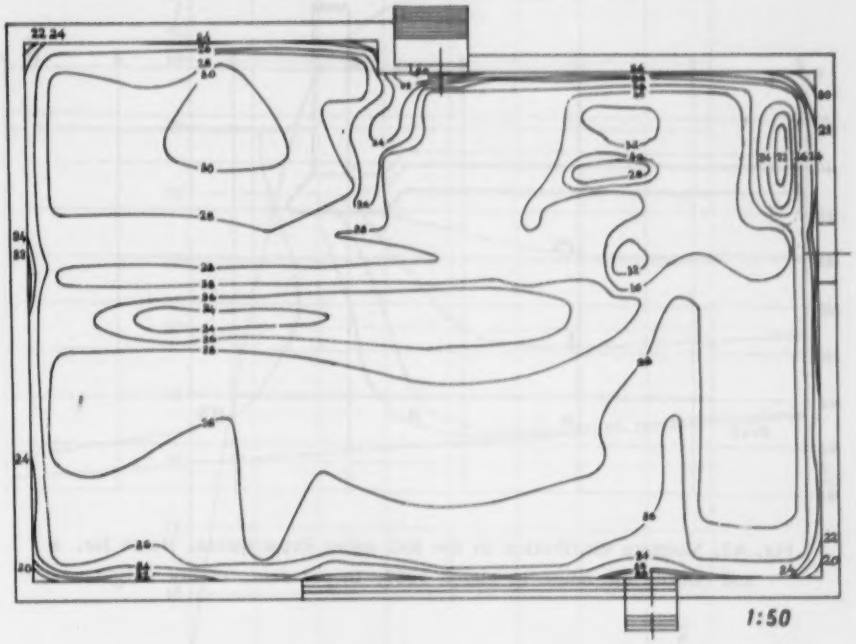

Fig. 42. Floor surface temperatures in Experimental House No. 1 when the mean of the outgoing and incoming heating water temperatures was T_m = 34,5 °C and the outdoor air temperature was t_o = - 11,2 °C.

No. 1 the maximum floor surface temperature exceeded 30 °C in places. However, the residents have not complained of discomfort, which is attributable to the presence of carpets and to other factors.

232. Moisture conditions in the soil in the sphere of influence of the slab

The moisture in the soil under the slab and at its edges has a remarkable influence on the thermal conductivity and thermal diffusivity of the soil and therefore also on the heat flow from the slab into ground.

During the warm season, the soil in the region around the edges of the slab and in the deep soil layers has an irregularly variable moisture content, depending

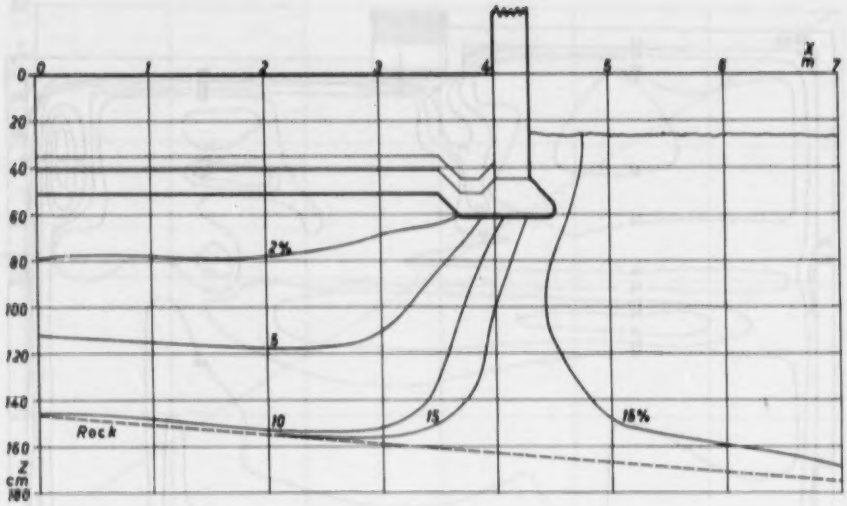

Fig. 43. Moisture distribution in the soil under Experimental House No. 1 and outside the house in March 1957, in per cent. of dry weight.

on the occurrence of rains. In the central regions of the slab and close to it, where the soil cannot be wetted by rainwater, the moisture content of the soil remains nearly constant throughout the year. According to performed measurements, the silt and fine sandy moraine soil under Experimental Houses No. 1 and 2 has a capillary height of 28 cm; therefore the moisture cannot rise to the upper layers by capillary suction. In the winter, when the ground is covered by snow, the soil attains a state of equilibrium with respect to its moisture content. Fig. 43 shows the loci of equal moisture content under Experimental House No. 1 and on its edges in the latter half of March, when the ground has a thick snow cover. The moisture contents found by measurement in three consecutive winters were very little different from the values entered in Fig. 43 close to the slab in its central region. At greater depth, i.e., close to the bedrock surface, and on the edges of the slab the moisture content in the autumn and spring may vary in the range of 10 ... 20 %, depending on the manner in which the snow melts and on the rainfall in the autumn. In the central region of the slab the soil has become drier, owing to the thermal energy supplied to the soil during the heating season. The moisture travels in the pore spaces of the soil in the form of water vapour from the warmer central regions to the cold edges, where its condensation increases the moisture content of the soil. This occurs because there is a higher saturated water vapour

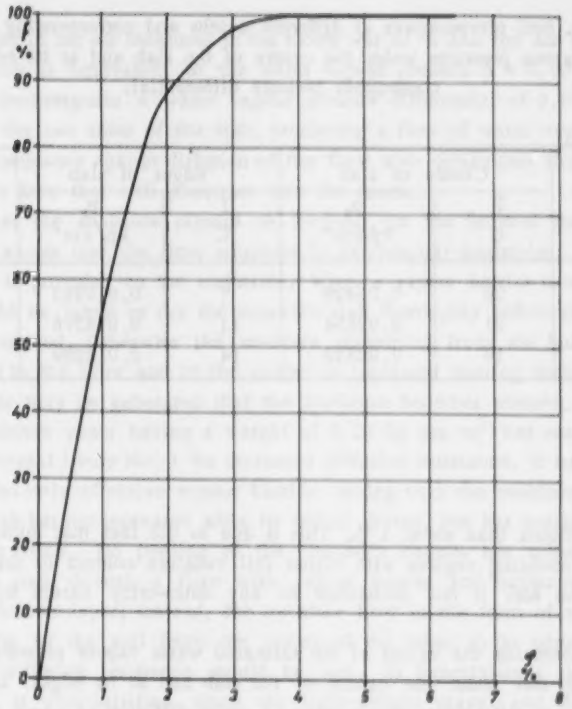

Fig. 44. Relative humidity of the air, f, in the pores of fine sand and silty till soil as a function of the moisture content of the soil, φ in per cent. of dry weight, at the temperature 20 °C.

pressure at each depth in the central regions of the slab, where the temperature is higher, than in the area on the edges.

The curve in Fig. 44 shows the relative humidity of the air in the pore spaces as a function of the soil's moisture content on the basis of measurements carried out with the soil type in question. As soon as the soil moisture exceeds 4 % (of the dry weight), the relative humidity of the air in the pore spaces is no longer dependent on the moisture content of the soil but it is constant, equalling 100 %. It could be established with the aid of a hair hygrometer lowered into the ground that the relative air humidity in the pore spaces was continuously 95 ... 100 % also under the centre of the slab although the measured moisture content of the

Table 1. Soil temperatures at different depths and corresponding saturated water vapour pressures under the centre of the slab and at its edges, and consequent pressure differential.

Depth, z cm	Centre of slab		Edges of slab		Δp_s kp/cm^2
	T °C	p_s kp/cm^2	T °C	p_s kp/cm^2	
0	26	0,03426	7	0,010210	0,02406
50	21	0,02534	11	0,013376	0,01197
100	19	0,02239	14	0,016289	0,00610

soil was no higher than about 1 %. This is due to the fact that humid air spreads from the surrounding regions with higher soil moisture content to the drier areas. However, the soil is not moistened to any noteworthy extent by this humid air.

Table 1 contains the values of the saturated water vapour pressure at different depths in the soil under the centre of the slab and at its edges, as well as the resultant pressure differential which tends to cause diffusion of water vapour in the direction from the central region to the edges.

As a consequence of its higher moisture content, the soil in the layers under the edges of the slab has considerably higher thermal conductivity than that under the centre of the slab. This increases the heat losses at the edges. The thermal conductivity of the soil in its different layers, taking into account its varying moisture content, can be found with the aid of the curve in Fig. 29 (p. 51).

Except by increasing the heat losses into ground, the high moisture content under the slab causes other detriments as well. Under conditions of varying moisture content the soil will alternately shrink and expand. The associated changes in volume are remarkable in magnitude in the case of fine-grained soil types, and they may give rise to subsidences and cause damage to the floor structure. Moisture may also result in other kinds of damage; for instance, linoleum carpets may become unstuck through the action of water vapour pressure under them.

According to our temperature measurements, the temperature in the soil under the experimental houses, at the centre of the slab and close to it, attains in midwinter a value of about 26 °C. When the air in the pore spaces of the soil has 100 % relative humidity, there will then be the pressure of saturated water vapour at this temperature, which is $p_s = 0,03426$ kp/cm^2. At a corresponding time, the

relative humidity of the air measured in the rooms was 30 % and the air temperature was 20 °C, which is equivalent to the water vapour pressure p = 0,0075 kp/cm^2. Under these circumstances a water vapour pressure differential of 0,0268 kp/cm^2 exists between the two sides of the slab, producing a flow of water vapour through the slab. The resistance against diffusion of the floor slab determines the magnitude of the moisture flow that will penetrate into the rooms.

In order that the moisture present in the soil, in the form of water vapour, may not gain access into the floor structure in detrimental quantities, a so-called vapour barrier is provided on the underside. When a vapour barrier is used, particular care should be taken to dry the concrete slab thoroughly before the linoleum carpets are attached. Otherwise the moisture remaining from the building phase will be trapped in the floor and in the course of increased heating such high water vapour pressures may be generated that the linoleum becomes unstuck.

Roofing bitumen paper having a weight of 2,25 kg per m^2 was used under the slab of Experimental House No. 1 for increased diffusion resistance. It has proved to furnish a sufficiently effective vapour barrier, seeing that the moisture content of the concrete slab has not increased after its initial drying, but has remainded below 3,5 % all the time. No peeling of the linoleum carpets has occurred. Water vapour cannot pass through a floor with vapour barrier and furthermore covered with a tight linoleum layer; instead, the moisture flow in the form of water vapour takes its course in the soil from the centre of the house to its edges, in which direction the diffusion resistance should be less. In Experimental House No. 1 this condition is well-satisfied, since the light-weight gravel and shingle layers obstruct diffusion in a very slight degree only.

The capillary ascent of water into the slab is prevented by interposing a coarse-grained gravel or shingle layer. Houses erected on floor slabs laid immediately on the ground should not be built on sites where it is impossible to prevent the entrance of water into the floor structure by leakage.

233. The heat quantities introduced in the house and escaping from it

We consider the case that thermal energy is delivered to the heating pipe system in the floor slab at a measured rate Q_c and that it is divided into a downward and upward heat flow, Q_1 and Q_2, respectively:

$$Q_c = Q_1 + Q_2 \tag{33}$$

The thermal flow Q_1 is consumed against the heat losses into ground and the losses directly into ambient air on the edges of the slab if the heating pipe level is above soil surface. The thermal flow Q_2 is utilized in heating the building. In the present work, the distribution of the thermal flow Q_c into upward and downward flow was determined with the aid of thermocouple measurements (cf. pp. 52 – 54).

In addition to the thermal energy Q_c delivered to the heating pipes, the building receives heat in the form of electrical energy at the rate Q_e and its residents supply heat at a certain rate, Q_r; furthermore there is the contribution of solar radiation in springtime. The thermal flow received in the form of radiation from the sun and other heat increments from occasional sources, which could not be determined, such as the heat from the fireplace or from the oven of the Sauna bath, have been summarized under the symbol Q_x. It should be noted that the effect of the two last-mentioned heat sources is highly transient and quite small. Q_x also accounts for extra heat losses, such as ventilation in greater than normal degree in the spring and autumn, and for the errors inherent in the results. We write

$$\Delta Q = Q_e + Q_r + Q_x \tag{34}$$

The heat flow downward from the slab and to its sides, Q_1, is consumed against the heat losses into ground, Q_G, and the losses at the edges of the slab directly into outdoor air, Q_A:

$$Q_1 = Q_G + Q_A \tag{35}$$

The upward heat flow Q_2 and the contribution of the other heat sources, ΔQ, is distributed among the heat losses by conduction through walls, roof, windows and doors, Q_C, and the losses caused by ventilation and air leakage, Q_V:

$$Q_2 + \Delta Q = Q_C + Q_V \tag{36}$$

The following equation is valid for the relationship between the rates of thermal

flow introduced in the house and escaping by way of the various losses:

$$Q_c + \Delta Q = Q_G + Q_A + Q_C + Q_V \qquad (37)$$

We shall consider these incoming and outgoing heat flow quantities on the basis of equation (37) in the following paragraph.

234. Distribution of the thermal energy $Q_c + \Delta Q$ introduced in the house

The thermal energy supplied to the house is shared by the different classes of losses in the manner shown by equation (37). These losses of different kinds, which will be discussed subsequently, and the heat quantities introduced in Experimental House No. 1 during the heating period 1956 – 1957 are shown by the graphs in Fig. 45, which also contains curves representing the variations in outdoor temperature and the difference between indoor and outdoor temperature during this time.

2341. Heat losses into ground

The thermal flow Q_G from the slab into ground is dependent on factors associated with the design of the slab, i.e., its area, shape, temperature and thermal lagging; on the other hand it is affected by the temperature, thermal conductivity and moisture content of the soil. Snow cover also has an influence on Q_G, as well as peculiarities of the terrain, such as concave or convex curvature of the soil surface and the presence of bedrock or ground water.

The thermal flow into ground has been calculated by the first formula of equations (32) for the central region of the slab, where $Q_G = Q_1$. The heat flow into ground at the edges of the slab has been calculated on the basis of the temperature gradients derived from the isotherm graphs and the thermal conductivity of the soil. In determining the values of the soil's thermal conductivity at different points under the slab, the moisture content has been taken into account (cf. Figs. 29, p. 51 and 43, p. 66).

The variation of the heat losses into ground, Q_G, during the heating period has been plotted in terms of daily means in Fig. 45 (Curve No. 3). Fig. 46 shows

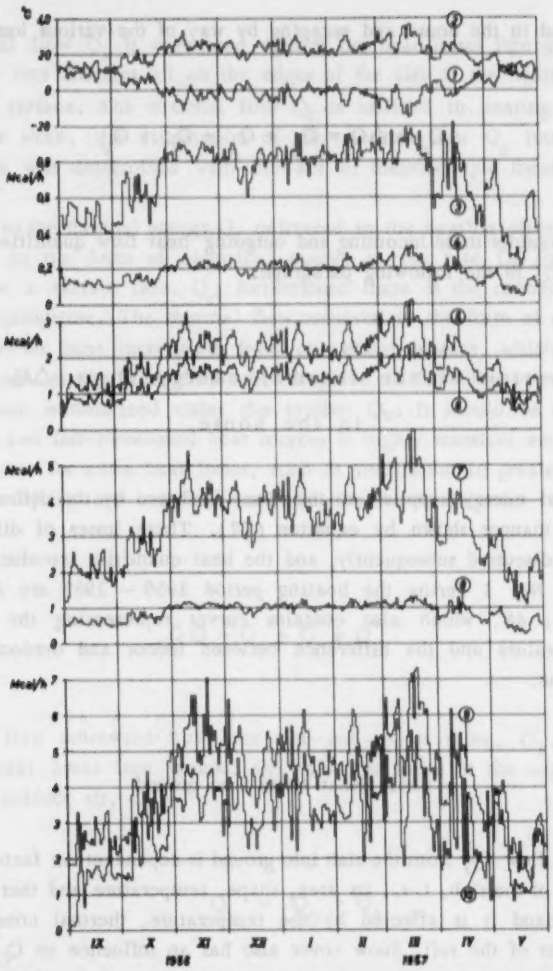

Fig. 45. Variations of outdoor air temperature, temperature difference between indoor and outdoor air, rates of introduction of thermal energy into Experimental House No. 1 and rates of heat losses from the house during the heating period 1956 — 1957.

1: Outdoor air temperature, t_o
2: Temperature difference between indoor and outdoor air, $t_i - t_o$.
3: Heat losses into ground, Q_G
4: Losses into outdoor air through the edges, Q_A.
5: Losses by conduction through the walls, Q_C
6: Losses by air leakage and ventilation, Q_V
7: Heat losses of the house, excluding the losses from the slab, $Q_C + Q_V$
8: Total losses from the floor slab structure, $Q_G + Q_A$
9: Total heat losses from the house, ΣQ.
10: Rate of supply of thermal energy from the heating central, Q_c.

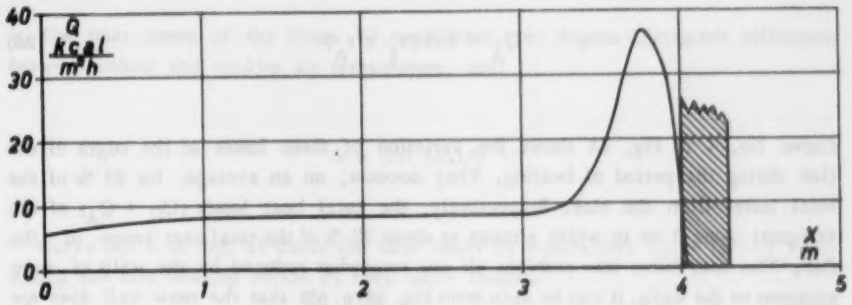

Fig. 46. Heat flow into ground in the cross section through the centre of the rectangular slab in Experimental House No. 1, on the basis of the isotherm graph derived from measurements on March 20th, 1957.

the thermal flow into ground at different points in the cross section of the slab on March 20th, 1957, at which time conditions were approximately stationary in the upper soil layers.

The heat losses into ground occur mainly at the edges of the slab. A zone 1 m in width from the interior surface of the outer walls already accounts for about 65 % of the entire heat flow from the slab into ground. The losses into ground from the central parts of the slab are comparatively high, owing to the closeness of bedrock (cf. Table 4, p. 86).

2342. Heat losses directly into outdoor air from the edges of the slab

The losses from the portions of the edges below ground level are included in the losses into ground, Q_G. The heat losses by dissipation into ambient air from the parts of the slab edge lying above soil have been calculated separately. The heat losses through the edging made of light-weight gravel concrete blocks have been calculated in the usual manner from the temperatures of the inner surface (t_1) and the outdoor temperatures (t_o), which were known from measurements. The said edging had a heat transmittance of $k = 0{,}75$ kcal/m^2 h $^\circ$C, and its area was $A = 7{,}75$ m^2. The heat losses through the edging directly into outdoor air are

$$Q_A = k A (t_1 - t_o) \qquad (38)$$

Curve No. 4 in Fig. 45 shows the variation of these losses at the edges of the slab during the period of heating. They account, on an average, for 20 % of the total losses from the slab. Respectively, the total heat losses ($Q_G + Q_A$) of the marginal zone 1 m in width amount to about 72 % of the total heat losses of the slab. The heat losses into ambient air are somewhat reduced by the walls of snow adjacent to the walls. It can be seen from Fig. 36 (p. 59) that the snow wall does not touch the wall of the house; melting produces within a few days an air gap, narrower on top and wider at its base, which gradually increases in width.

2343. Heat losses by conduction

The heat losses of the building by conduction through the walls directly into outdoor air (excluding the heat losses through the floor) were calculated separately for each component surface A_ν per one degree Centigrade temperature difference between indoor and outdoor air. The thermal conductance of the house is:

$$G = \Sigma k_\nu A_\nu \qquad (39)$$

The following heat transmittance coefficients k, calculated on the basis of the design, have been used for the various structural parts:

- Walls of light-weight concrete (dry volume weight 500 kg/ m^3), plaster-covered on both sides; thickness 25 cm and average moisture content 5 %	k = 0,48 kcal/m^2 h ^oC
- Uncaulked windows with double panes	2,5 "
- Uncaulked windows with triple panes	1,8 "
- Double outer door with small entry	3,0 "
" " " " 8 cm air space	4,0 "
- Ceiling and roof, measured value (cf. p. 54)	0,177 "

The heat losses of the house by conduction, per degree Centigrade difference between indoor and outdoor air temperature, are:

$$G = 100 \text{ kcal/h } ^{\circ}C$$

Curve No. 5 in Fig. 45 shows the heat losses by conduction, $Q_C = (t_i - t_o)\, G$, during the said heating period by their daily means.

2344. Heat losses due to air leakage and ventilation

By measurements carried out in the manner described at an earlier place (p. 55), the following air change coefficients were found for Experimental House No. 1:

$n = 1,06$ times per hour for wind velocities $2,7 \dots 3,7$ m/s and at the outdoor air temperature $+6,1\ ^{\circ}C$,

$n = 0,69$ times per hour for wind velocity about 0 m/s and at the outdoor air temperature $-10,6\ ^{\circ}C$.

The value $n = 0,8$ times per hour was used for the average air change coefficient; this figure resulted from consideration of the average wind conditions. The heat losses caused by air leakage and ventilation, per degree Centigrade temperature difference between indoor and outdoor air, G_V, are:

$$G_V = n\, V \varrho c_p \tag{40}$$

Substituting:

Air change coefficient $n = 0,8$ times per hour,

Air space volume of Experimental House No. 1 $V = 284\ m^3$,

Specific heat of air ($t = 20\ ^{\circ}C$) at constant pressure $\varrho c_p = 0,29$ kcal/m^3 $^{\circ}C$,

we find

$$G_V = 66,0 \text{ kcal/h } ^{\circ}C$$

The mean rate of heat losses by air leakage and ventilation during each

24-hour day has been calculated by multiplying G_v by the difference between the mean indoor and outdoor air temperatures of that day:

$$Q_V = (t_i - t_o)\, G_v$$

Curve No. 6 in Fig. 45 represents the heat losses by air leakage and ventilation, Q_V, as daily means during the heating period 1956 – 1957.

2345. Classes of the heat losses

The heat losses of Experimental House No. 1 excluding those from the floor slab structure, i.e., the sum $Q_C + Q_V$, have been plotted as curve No. 7 and the total losses from the slab as curve No. 8 in Fig. 45.

The first-mentioned losses follow closely the changes in outdoor air temperature, but the losses of the second class have been approximately about 1000 kcal/h from the beginning of November to the end of March, that is, during the time when the sides of the house were snow-covered.

In addition to the temperature difference between indoor and outdoor air, also wind, sunshine and rainfall exert an influence on the heat losses of the part of the house which is above ground.

The heat losses from the floor slab structure in contact with the soil are determined by the thermal conditions in the soil, the abundance of snow and the type of the soil and its moisture content. It has been noted before (p. 14) that the temperatures in undisturbed soil are constantly close to 0 °C at the surface of snow-covered ground; with increasing depth, the temperature increases to a constant value of about 7,2 °C. It is therefore readily understandable that the heat losses from the slab into ground should vary very little during the time when the ground is under snow. In the autumn, when heating is commenced, the heat losses into ground immediately rise to about one half of the wintertime value because the soil starts to store heat. In the spring, when heating is reduced, the available heat stores in the soil cause lessening of the heat losses from the slab into ground.

Curves No. 9 and 10 in the lower part of Fig. 45 represent the total heat losses of Experimental House No. 1 and the rate of heat delivery to the house from the heating central, ΣQ and Q_c, respectively. Their difference ΔQ is equivalent to the heating power received in other ways. This extra heat supply satisfies about 23 % of the annual heat requirements of the house.

235. Thermal balance of Experimental House No. 1

The annual balance for Experimental House No. 1 with regard to thermal energy can be seen in Tables 2 and 3 for the heating periods 1956 – 1957 and 1957 – 1958. The tables show the monthly totals for the heat loss groups Q_C, Q_V, Q_A and Q_G and their sum, ΣQ, the heat quantities Q_c supplied to the floor slab during the corresponding times, as well as the figures for the entire heating period. Also included is the distribution of the difference $\Delta Q = \Sigma Q - Q_c$ between the total heat losses and the heat quantity indicated by the calorie counter by the groups of electrically produced heat Q_e, heat dissipation of the residents Q_r and other heat sources Q_x. The heat generated by electrical means was calculated from the readings of the household's electricity meter, assuming the consumed electrical energy to be converted into heat in its entirety. The calculations of the heat dissipation by the residents (two adults; two children, aged 10 and 3) was based on the assumption that an adult person releases 80 kcal/h on an average, and a child 40 kcal/h. The figures in Tables 2 and 3 were obtained on these premises by taking into account the time spent indoors by each person. The third component Q_x of the difference ΔQ (cf. eq. 34, p. 70) includes the heat received in the form of solar radiation, which is an essential factor in the spring months, and the heat from other occasional sources of heat (fireplace, Sauna bath oven). Furthermore, it has a negative constituent, namely, the heat lost by extra ventilation. It is due to this circumstance that Q_x obtains negative values in September and May. The lumped errors incurred in the calculations for the different classes of heat losses are also contained in the quantity Q_x. One of the recognized sources of error is the snow cover existing from time to time on the roof, which reduces the heat losses by conduction through ceiling and roof. For instance, the heat losses of the house by conduction are about 3 % lower when there is a snow layer (λ = 0,1 kcal/m h °C) of 20 cm thickness on the roof. However, the roof of the experimental house was only transiently covered by snow, its sheet aluminium surface shedding the snow as soon as the weather became warmer.

In Experimental House No. 1, having walls and roof structure with fairly good thermal insulating characteristics, the heat losses by thermal flow into the ground accounted for 16,9 and 16,6 % of the total heat losses of the house in the heating periods of 1956 – 1957 and 1957 – 1958, respectively. Heat losses equivalent to those through the floor of this experimental house during the time January 1st to March 31st will be encountered in a house built in the conventional manner and provided with basement or crawling space, and heated with radiators, when its floor has a thermal transmittance coefficient k = 0,51 kcal/m² h °C and the temperature in its basement or crawling space is 5 °C.

Table 2. Thermal energy balance of Experimental House No. 1 for the heating period 1956 – 1957

Month	Mean temperature, °C	Heat losses, Mcal					Supplied heat quantity, Mcal		Contribution of additional heat sources, ΔQ, Mcal		
		Q_C	Q_V	Q_A	Q_G	ΣQ	Q_c	$\Delta Q = \Sigma Q - Q_c$	Q_e	Q_t	Q_x
September	9,6	799,3	568,8	79,2	345,6	1792,8	1173,6	619,2	396,0	144,0	79,2
October	6,1	1242,5	825,8	104,2	461,3	2633,8	2060,9	572,8	409,2	148,8	14,8
November	-4,5	1778,4	1188,0	158,4	612,0	3736,8	3016,8	720,0	396,0	144,0	18,0
December	-5,3	1659,1	1108,6	148,8	587,8	3504,2	2767,7	736,7	409,2	148,8	178,6
January	-0,5	1718,6	1130,9	156,2	617,5	3623,3	2812,3	810,9	326,6	148,8	335,6
February	-1,6	1585,9	1041,6	141,1	571,2	3339,8	2661,1	678,7	275,5	134,4	268,8
March	-5,4	2053,4	1354,1	178,6	647,3	4233,4	3132,2	1101,1	284,2	148,8	668,1
April	2,3	1389,6	914,4	115,2	489,6	2908,8	2124,0	784,8	252,0	144,0	388,8
May	8,8	930,0	610,0	81,8	357,1	1979,0	1644,2	334,8	251,5	148,8	65,5
Entire heating period	1,1	13156,8	8742,2	1163,5	4689,4	27751,9	21392,8	6359,0	3000,2	1310,4	1886,4
Per cent.	contribution	47,4	31,5	4,2	16,9	100	77,1	22,9	10,8	4,7	7,4

Table 3. Thermal energy balance of Experimental House No. 1 for the heating period 1957 – 1958

Month	Mean temperature, °C	Heat losses, Mcal					Supplied heat quantity, Mcal		Contribution of additional heat sources, ΔQ, Mcal		
		Q_C	Q_V	Q_A	Q_G	ΣQ	Q_c	$\Delta Q = \Sigma Q - Q_c$	Q_e	Q_r	Q_x
September	10,6	762,4	542,6	70,8	317,7	1693,5	1272,2	421,3	246,2	180,0	-4,9
October	6,9	1153,0	766,4	92,1	405,0	2416,5	1852,6	563,9	275,3	186,0	102,6
November	2,1	1394,3	931,4	123,8	543,0	2992,5	2426,4	566,1	288,0	180,0	98,1
December	-0,7	1607,7	1074,2	138,4	597,5	3417,8	2894,2	523,6	297,6	171,1	54,9
January	-6,8	2113,9	1391,0	193,6	706,9	4405,4	3370,3	1035,1	267,8	186,0	581,3
February	-8,4	2063,3	1355,1	179,8	629,5	4227,7	3077,8	1149,9	215,0	168,0	766,9
March	-6,1	2108,9	1390,6	181,2	658,2	4338,9	2797,4	1541,5	200,9	186,0	1154,6
April	1,3	988,0	650,1	81,9	360,4	2080,4	1627,2	453,2	165,6	180,0	107,6
May	7,7	552,4	362,4	57,2	251,5	1223,5	1101,1	122,4	163,7	186,0	-227,3
Entire heating period	0,7	12743,9	8463,8	1118,8	4469,7	26796,2	20419,2	6377,0	2120,1	1623,1	2633,8
Per cent. contribution		47,6	31,6	4,2	16,6	100	76,2	23,8	7,9	6,1	9,8

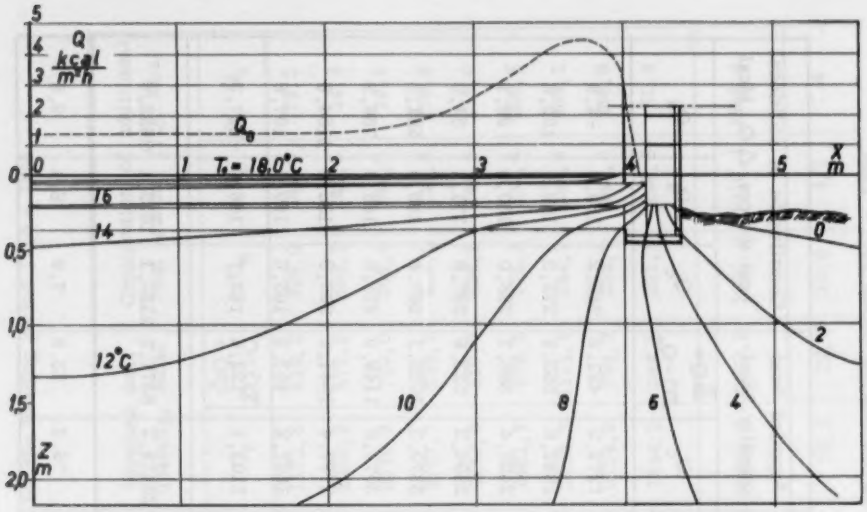

Fig. 47. Temperature distribution field in the soil in section A-A (Fig. 27, p. 49) through the marginal region of the unheated rectangular floor slab in Experimental House No. 4 on February 19th, 1959.

236. Unheated floor

Unheated floor structures placed immediately on the ground are represented by Experimental House No. 4 in our investigation, i.e., the secondary school building of Tapiola. The design of its floor structure and the constitution of the soil under the slab, as well as the location of thermocouples and heat flow meters have been described in Fig. 27 (p. 49). The sand layer in the soil under the slab had a wintertime moisture content of 10 ... 12 % in the central region of the slab and 14 ... 16 % in the area under its edges. According to Saare's [17] investigations, the value λ = 1,2 kcal/m h °C is found for its thermal conductivity.

Fig. 47 shows the temperature distribution field in the soil under the rectangular part of the slab in the vertical section A-A (Fig. 27) in the winter (February 19th, 1959). The average surface temperature of the slab was 18 °C.

In the summer the measured temperatures (June 2nd, 1959) at corresponding points in the central region of the slab were identical with those observed in the winter within about 1 °C (Fig. 47). The temperatures in the upper soil layers close to the edges of the slab were considerably colder in wintertime, but the difference at 1 ... 2 m depth is only about 2 °C.

The thermal flow from this unheated slab into ground was measured with

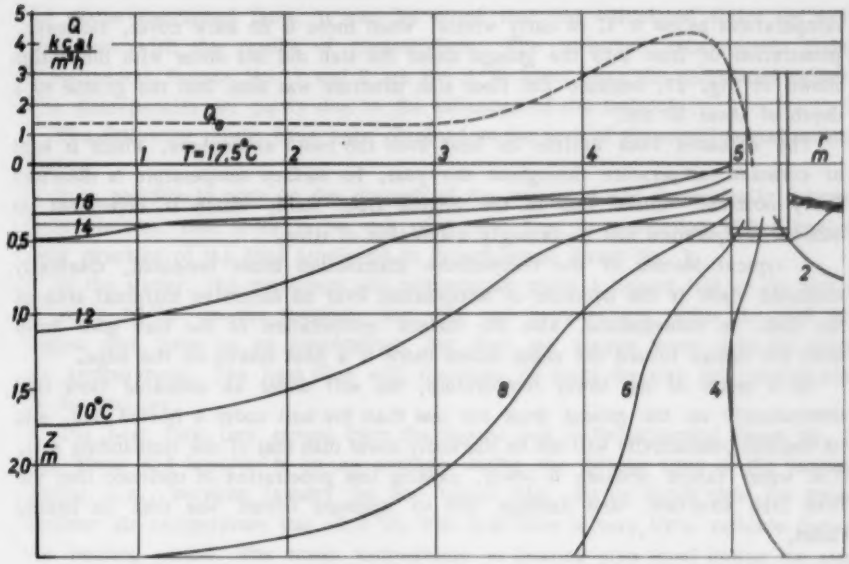

Fig. 48. Temperature distribution field in the soil in section B-B (Fig. 27, p. 49) of the unheated hexagonal floor slab in Experimental House No. 4 on February 19th, 1957.

a heat flow meter in the central region of the slab and on its edge. In the winter of 1959 (February 1st to March 31st) it was on an average 1,3 kcal/m^2h in the central region and 4,2 kcal/m^2 h on the edge. The mean indoor temperature during this time was 18,7 °C, the outdoor air temperature 0,5 °C and the soil surface temperature 0 °C. It can be seen that the losses into ground from a large, well-insulated unheated slab are quite small per unit area.

The temperature distribution field existing under the slab having the shape of a regular hexagon, in the section B-B (Fig. 27, p. 49), on February 19th, 1959, is shown in Fig. 48. The temperature gradient and the thermal flow from the slab into ground at the corners have somewhat greater magnitude than those of the parts adjacent to a straight wall (cf. Fig. 47).

Owing to the lower slab temperature, the soil temperatures under the unheated slab are considerably lower than under a heated slab. In midwinter, when the heated slab has a mean temperature of T_0 = 33 °C at the heating pipe level and the surface temperature of the unheated slab is 17,5 °C on an average, there is a difference of 6 ... 10 °C in the 0 to 1 m soil layer in the central region of the slab. In the soil outside the lagging at the edges of the unheated slab there are

temperatures below 0 °C in early winter, when there is no snow cover. However, penetration of frost into the ground under the slab did not occur with the design shown in Fig. 27, because the floor slab structure was sunk into the ground to a depth of about 30 cm.

The unheated floor derives its heat from the room atmosphere, which is kept at constant temperature throughout the year. Its surface temperature is therefore fairly constant, unlike that of the heated floor slabs, which is dependent on outdoor temperature and accordingly a function of time.

A typical feature of the temperature distribution under unheated, thermally insulated slabs is the decrease of temperature over an extensive marginal area of the slab. In consequence, also the surface temperature of the slab goes down from the centre toward the edges unless there is a heat source on the edge.

As a result of the lower temperature, the soil under an unheated floor laid immediately on the ground dries out less than the soil under a heated slab, and its thermal conductivity will not be markedly lower than that of the surrounding soil. The water vapour pressure is lower, causing less penetration of moisture into the floor slab structure, and damage due to moisture occurs less than in heated floors.

24. INFERENCES FROM THE EXPERIMENTAL RESULTS

The soil type is mostly greatly variable in Finland. This is also reflected by the fact that the experimental houses concerned in this investigation could not be erected on homogenous, isotropic soil, as would have been consistent with the theoretical calculations. The theoretically derived temperature distribution fields cannot be in full agreement with the results of measurements for this reason.

When calculating the temperature field under an uninsulated concrete slab, the mean temperatures in the heating tube plane can be used as the temperature T_o, because the thermal conductivity of a concrete slab reinforced with steel does not differ considerably from the thermal conductivity values of the soil. When calculating the temperature field of an insulated slab, the temperature of the soil under the insulation has to be taken as a starting-point. The theoretical assumptions are best valid in the case of calculations concerning the soil temperatures and the heat losses into ground under greenhouses and athletics halls, for instance.

On comparing the measured temperature distributions and the theoretically calculated stationary fields one should also keep in mind the fact that the field existing in the soil does not attain a steady state during the heating period and that the temperatures in deeper layers of the theoretically derived field will therefore be higher than those encountered in actual practice. In a non-stationary field, again, deviations are produced, e.g., by the fact that the thermal diffusivity is not constant throughout the heating period. Taking into consideration

the above-mentioned facts, it can be noted that the calculated fields are well consistent with the results of measurement. The measured temperatures under Experimental Houses No. 1 and 2 are 2 ... 3 °C lower than the calculated values. The discrepancies are partly due to the influence of the bedrock.

The following results have been obtained on the basis of the experimental investigation:

- It is possible to note on the strength of the measured and theoretically derived field pictures, that there is no risk of ground frost in connection with a heated floor structure of the kind employed in Experimental House No. 1.
- In the winter, the maximum soil temperature under a heated slab in its centre is about 30 °C. In the summer, too, the soil temperatures under the slab are higher than those in its surroundings, but they are always lower than the room air temperatures. The heat flow will therefore be from the slab into ground also in the summer.
- The heat flow into ground from the heated slab in Experimental House No. 1 was about 8,4 kcal/m^2 h on an average during the coldest time of the heating period, i.e., between January 1st and March 31st, during which time the mean outdoor air temperature was -3,2 °C. The heat flow is very little variable during the heating period. The mean temperature at heating pipe level during the said time was 29,5 °C.
- The above-mentioned thermal flow is consistent with an apparent heat transmittance $k_a = \frac{8,4}{29,5}$ kcal/m^2 h °C = 0,284 kcal/m^2 h °C, as calculated for the existing temperature difference between heating pipe level and surrounding soil surface covered with snow. The apparent heat transmittance k_a is lower than that calculated on the basis of the actual thermal resistance of the soil, because it also includes the effect of the heat flow upward from the ground (cf. equation 51).
- From an unheated, insulated floor slab (Experimental House No. 4; Fig. 47) there occurs a heat flow from the slab into ground, in midwinter when the ground lies under a fairly thick snow cover, of 1,3 kcal/m^2 h in the central region of the slab and 4,2 kcal/m^2 h in its marginal parts.
- The heat losses into ground from the heated floor slab in a well-insulated house on a moraine hillock (Experimental House No. 1) were about 16 % of the entire heat losses of the house during the heating period. On ground which does not freeze, these losses can be reduced, and some reduction can also be achieved on ground liable to frost, by increasing the insulation at the centre to the slab.
- The heat losses from the edges of the slab directly into ambient air were 20 % of the total losses from the slab during the same period. It is resonable to reduce the losses of this group by improving the insulation on the edges.
- The heated slab causes considerable drying of the underlying soil, so that the heat losses into ground are decreased. The drying of the soil continues to some extent during subsequent heating periods.

- Carpets cause an increase of the heat losses into ground because the temperatures of floor surface and heating pipe level have to be increased in order to achieve adequate delivery of thermal energy into the room.
- A vapour barrier under the slab is necessary in the case of a heated slab also when the soil is comparatively dry, because the air in the pore spaces of the soil always has a relative humidity close to 100 % in actual practice.
- Floor heating is a pleasant and comfortable mode of heating provided that the house is insulated well enough to make sure that floor surface temperatures exeeding 27 ... 30 °C will not have to be used save in exeptional circumstances.
- When a house is heated by floor heating, the heating elements may not be shut off in any one of the rooms, on account of the risk of ground frost and the thermal stresses produced in the slab if this is done. If unheated spaces or spaces heated with radiators are connected with a building furnished with floor heating, an expansion joint has to be provided. It is also necessary to arrange for flexibility of the pipe connections.

3. PRACTICAL CALCULATIONS OF THE HEAT LOSSES FROM THE FLOOR SLAB INTO GROUND, AND DIMENSIONING OF THE THERMAL LAGGING

31. UNINSULATED SLAB

For calculations of the heat losses from an uninsulated slab into ground, the equations for the thermal flow presented in the first section of this work are valid as such. This third section contains tables giving numerical values for different types of slabs, calculated by the said equations with the aid of an electronic computer.

311. Rectangular slab

The heat quantities flowing from a rectangular slab and from its different zones (Fig. 9, p. 25) into homogeneous soil, calculated by equation (5), are compiled in Table 4.

Moreover, Table 4 contains the mean resistance of the soil against thermal flow $m_{o\nu}$ for each zone ν calculated by the formula

$$m_{o\nu} = \frac{1}{k_{o\nu}} = \frac{A_\nu T_o}{Q_{30\nu}} \tag{41}$$

The table can be used for a mean slab temperature other than $T_o = 30\ ^oC$; it is only necessary to multiply the heat flow values $Q_{30\nu}$ for the different zones by the factor T/T_o. Similarly, multiplication of $Q_{30\nu}$ by λ/λ_o takes care of the

Table 4. Heat flow from a rectangular slab (a = 4,0 m, a'-a = 0,50 m; b = 6,2 m, b'-b = 0,78 m) into homogeneous soil (λ_o = 1 kcal/m h °C) when T_o = 30 °C and when the soil temperature without the field of the slab is 0 °C.

Zone No.	Area of the zone A_ν		Heat flow $Q_{30\nu}$			Thermal resistance $m_{o\nu}$
	m^2	%	kcal/h	%	kcal/m^2h	m^2h °C/kcal
I	19,4	19,6	525,2	43,1	27,07	1,108
II	17,4	17,5	244,7	20,0	14,06	2,134
III	15,4	15,5	158,5	13,0	10,29	2,915
IV	47,0	47,4	290,8	23,9	6,19	4,847
Entire slab	99,2	100	1219,2	100	12,29	2,441

required conversion if the thermal conductivity of the soil differs from λ_o = 1 kcal/m h °C:

$$Q_\nu = \frac{Q_{30\nu}}{T_o \lambda_o} T\lambda \qquad (42)$$

For another thermal conductivity of the soil, the thermal resistance is derived from the $m_{o\nu}$ values in Table 4 by multiplication by the factor λ_o/λ:

$$m_\nu = \frac{\lambda_o}{\lambda} m_{o\nu} \qquad (43)$$

The data contained in Table 4 can be used in calculating the thermal flow for a series of homothetic slabs of different sizes, obtained from the original slab when its dimensions a, a', b and b' are all multiplied by the same conversion factor K. They also furnish practical approximations if the shape of the slab is slightly different from that of the reference slab. The procedure has been outlined on p. 43.

A temperature drop as described in boundary conditions in the region beyond the edges of the slab exists in actual practice if the outer edge of the slab is lagged in a given manner. At the lagging on the edges the vertical component of the heat flow, which represents the flow directed into the ground, has downward direction at soil surface as far as about the middle of the lagging at the edges (cf. Figs. 6, 7 and

8, pp. 22 – 24). On the outer edge of the lagging the vertical component of the heat flow points upward. The limit at which the change in direction of the heat flow occurs can be seen approximately in Figs. 6, 7 and 8. A complicated analytical transcendental expression would be required to describe it and it is a highly cumbersome task to evaluate the heat flow integral (5) exactly to the said limit. The integral is therefore extended to the centreline of the lagging at the edge, i.e., to the values $\frac{a + a'}{2}$ and $\frac{b + b'}{2}$, respectively. When calculated in this manner, the heat flow from the lagging at the edges of a slab of the kind concerned in Table 4 into the soil (λ_o = 1 kcal/m h °C) will be 139,2 kcal/h and that from the entire slab, including its marginal area, 1358,4 kcal/h. The marginal zone having the width $\frac{a' - a}{2}$ or $\frac{b' - b}{2}$ and the area A_E = 12,8 m^2 gives off a mean heat flow per unit area of 10,88 kcal/m^2h.

In the marginal zone the equations (42) and (43) are valid, and moreover equation (41) if $T_o/2$ is written in this equation instead of T_o.

The upward heat flow from the ground in wintertime has, at the surface (z = 0), opposite direction to that from the slab into ground and it has to be deducted from the downward flow from the slab. The flow density of the upward heat flow q_o is obtained as the product of the temperature gradient in undisturbed free soil (Figs. 1 – 5) and the thermal conductivity of the soil (Table 8, p. 95).

312. Slabs having the shape of a narrow strip

The heat flow per unit area from an uninsulated slab having the shape of a narrow strip (2a = 8,0 m, a' - a = 0,50 m) into homogeneous soil at the cross section of the slab calculated by equation (7), has been shown in Fig. 10 for T_o = 30 °C and for the thermal conductivity λ_o = 1 kcal/m h °C of the soil. The heat flow density from the central region of the slab into ground has a very small magnitude but it increases strongly towards the edge of the slab. At the lagging on the edge of the slab the temperature declines and the heat flow becomes zero approximately at the middle of the lagging. Beyond this point there is an upward heat flow from the ground, which attains its maximum at the outer edge of the lagging, after which it abates again to zero with increasing distance from this edge. The area above the x axis enclosed by the curve representing the heat flow density gives a measure of the thermal energy given off by the slab into the ground. The total heat flow from the slab into the ground and the corresponding values for the different zones, per unit length, have been calculated by equation (7) and compiled in Table 5. The slab having the shape of a narrow strip was divided into zones in conformity with the zone division of the rectangular slab as reflected by its cross section (Fig. 9, p. 25). Table 5 also contains the mean thermal resistance values of the soil for the different zones and for the entire slab.

Table 5. Heat flow from a slab having the shape of a narrow strip (2a = 8,0 m, a' - a = 0,50 m) into homogeneous soil (λ_o = 1 kcal/m h °C), per unit length (m), when T_o = 30 °C and when the soil temperature without the field of the slab is 0 °C.

Zone No.	Area of the zone A_ν		Heat flow $Q_{30\nu}$			Thermal resistance $m_{o\nu}$
	m^2	%	kcal/h	%	kcal/m^2h	m^2h °C/kcal
I	1	12,5	30,93	41,4	30,93	0,970
II	1	12,5	9,24	12,4	9,24	3,247
III	1	12,5	8,21	11,0	8,21	3,654
IV	5	62,5	26,33	35,2	5,27	5,696
Entire slab	8	100	74,71	100	9,33	3,215

The table can be used for other mean temperatures T of the slab and other thermal conductivities λ of the soil by means of the conversion equations (42) and (43); the heat flow upward from the soil has then to be deducted from the values obtained in this manner. For slabs of different width, the values in Table 5 are converted in the manner outlined on p. 43.

The heat flow into ground at the lagging on the edge of the slab, as calculated for the width from its inner edge to its centreline, is 11,35 kcal/h, and the upward heat flow from the ground through the area between the centreline and the outer edge of the lagging is 12,12 kcal/h, both per unit length (m). The boundary at which the heat flow reverses its direction is a straight line lying slightly inside the centreline.

The combined heat flow into ground from the slab in question and from its marginal zone is 86,06 kcal/h per one metre of slab length.

If the width of the marginal zone, which was a' - a = 0,50 m, is reduced to, say, a' - a = 0,30 m (i.e., by 40 %), the heat flow from the slab increases from 74,71 kcal/h to 81,98 kcal/h, i.e., by 9,7 %. It is seen that the total heat flow from the slab into ground is not very rigidly controlled by the rate of descent of the temperature in the marginal lagging of the slab. The same applies to rectangular and circular slabs.

313. Circular slab

The thermal flow from an uninsulated, circular slab (R_o = 4,73 m, R = 5,26 m, T_o = 30 °C, μ = 0,9) and from its different zones into homogeneous soil of ther-

mal conductivity λ_0 = 1 kcal/m h °C and the corresponding values of the thermal resistance imposed by the soil have been compiled in Table 6. The zones are rings of 0,50 m width, starting at the edge of the slab with R_0 = 4,73 m. The heat flow values for the different zones were calculated by equation (24).

Table 6. Heat flow from a circular slab (R_0 = 4,73 m, μ = 0,9) into homogeneous soil (λ_0 = 1 kcal/m h °C) when T_0 = 30 °C and when the soil temperature without the field of the slab is 0 °C.

Zone No.	Area of the zone A_ν		Heat flow $Q_{30\nu}$			Thermal resistance $m_{0\nu}$
	m^2	%	kcal/h	%	kcal/m^2h	m^2h °C/kcal
I	14,08	20,0	579	59,6	41,11	0,730
II	12,50	17,8	155	16,0	12,36	2,427
III	10,93	15,6	81	8,3	7,41	4,049
IV	32,77	46,6	156	16,1	4,75	6,309
Entire slab	70,25	100	971	100	13,82	2,171

Table 6 can be adapted for use at other mean temperatures of the slab and other thermal conductivities of the soil by conversion of the values in accordance with equations (42) and (43). Conversion for different values of the radius is done by the same principle as for rectangular slabs and slabs having the shape of a narrow strip (p. 43). The upward heat flow from the ground is taken into account in the same manner as in the preceding instances.

The heat flow into ground from the circular slab concerned in Table 6, calculated for the portion of its marginal lagging from its inner edge to its centreline, is 146 kcal/h, i.e., 17,68 kcal/m^2h.

32. INSULATED SLAB

321. Thermal lagging of the floor slab

In instances where the thermal resistance inherent in the soil is not in itself sufficient to keep the heat losses into ground small enough, thermal insulation of the floor slab has to be provided.

Table 7. Heat transmittance coefficients k_o of the soil under different kinds of slabs, for thermal conductivities of the soil $\lambda = 0,5 \cdots 3,0$ kcal/mh°C.

Type of slab	Heat transmittance coeff. k_o, kcal/m²h°C					
	$\lambda = 0,5$	$\lambda = 1,0$	$\lambda = 1,5$	$\lambda = 2,0$	$\lambda = 2,5$	$\lambda = 3,0$
Rectangle,						
b/a = 1,55						
(a'-a)/a = 0,125						
A_o = 100 m²	0,204	0,408	0,612	0,816	1,020	1,224
A = 200 m²	0,144	0,289	0,433	0,577	0,721	0,866
Narrow strip,						
(a'-a)/a = 0,125						
2a = 8 m	0,156	0,311	0,467	0,622	0,778	0,933
2a = 16 m	0,078	0,156	0,233	0,311	0,389	0,467
Circle, $\mu = 0,9$						
R_o = 5,64 m, A_o = 100 m²	0,193	0,386	0,579	0,772	0,965	1,158
R_o = 7,98 m, A = 200 m²	0,137	0,273	0,410	0,546	0,683	0,819

Table 7 gives the heat transmittance coefficients k_o of the soil under floor slabs having the shape of a rectangle, a narrow strip and a circle, respectively, for thermal conductivities of the soil in the range $\lambda = 0,5 \cdots 3,0$ kcal/m h °C. The heat transmittance coefficient has been calculated for slabs of two sizes, 100 m² and 200 m², in each instance, and it has been given as calculated for the soil layers between the lower surface of the warm slab and the soil surface in the area surrounding the house, on the basis of the total thermal resistance of the soil found in Table 4, 5 and 6. A change in size of the slab from that concerned in Table 7 changes the heat transmittance of the soil under the floor structure proportionally to the inverse value of the dimensional ratio K:

$$k' = \frac{1}{K} k_o \qquad (44)$$

For an unheated floor structure, having a temperature difference of about 18 °C between the floor surface and the snow-covered soil surrounding the house, the heat loss figure of 5 kcal/m² h can be considered reasonable. This implies a mean heat transmittance k = 0,28 kcal/m²h°C of the floor. It is seen from Table 7 that the heat transmittance of the soil under a rectangular slab of 100 m² is $k \leq 0,28$ kcal/m²h°C for thermal conductivities of the soil not higher than $\lambda = 0,679$ kcal/mh°C. This limit is surpassed in all other cases except in very dry sand or clay soil [17].

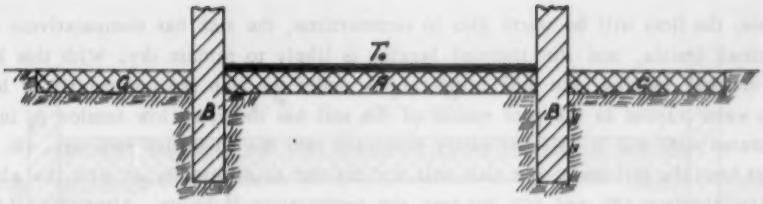

Fig. 49. Three typical lagging methods employed for thermal insulation of floor slabs: (A) Lagging under the slab, (B) Lagging of the foundation, (C) Lagging provided at soil surface outside the slab.

When the thermal conductivity of the soil exceeds the said value, the slab has to be insulated in order to keep the floor surface temperature at sufficient height and to reduce the heat losses. In all cases lagging has to be provided at least on the edges of the slab so that a sufficiently high floor surface temperature might be achieved also on the edges (insulation by alternative B, Fig. 49).

Lagging of a heated slab is necessary only in view of reducing its heat losses. 10 kcal/m^2h can be considered a reasonable magnitude for the heat losses from a rectangular floor slab having T_o = 36 °C at the level of the heating pipes; this corresponds to the heat losses into ground during very cold weather encountered in Experimental House No. 1. These values are consistent with the k_o and λ values found in the foregoing for an unheated slab. It is seen that a heated slab has to be lagged as a rule in order to reduce its heat losses.

As the thermal conductivity is strongly dependent on the moisture content of the soil, drying of the soil by underdrainage is in itself already an efficient expedient towards higher thermal insulation. For instance, if the moisture content of till soil is lowered from 12 % to 4 %, its thermal conductivity is reduced from λ = 1,90 kcal/mh°C to λ = 1,35 kcal/mh°C. This is equivalent to lowering the heat transmittance coefficient of the soil under a rectangular slab of 100 m^2 size from k_o = 0,775 kcal/m^2h°C to k_o = 0,551 kcal/m^2h°C. The heated slab contributes to the process of drying the soil, as has been observed before (cf. Fig. 43, p. 66). However, high ground water table and capillary suction, if the soil has a great capillary height, tend to offset the drying effect of the slab.

Thermal insulation of the floor slab can be achieved in one of three different ways, namely, (A) with the aid of a lagging course under the lower surface of the slab, (B) by insulating the foundations, and (C) by insulating the soil surface on the outer edges of the building (Fig. 49).

Thermal insulation according to alternative (A) is easy to apply; it requires only little excavation work and it is also suitable when the building is erected on rock. Moreover, this method of insulation is advantageous from the viewpoint of heat

losses; the floor will be warm also in summertime, the slab has comparatively low thermal inertia, and the thermal lagging is likely to remain dry. With this kind of thermal lagging, the soil temperatures under the slab are comparatively low. The water vapour in the pore spaces of the soil has therefore low tension p_s in its saturated state and it will not easily penetrate into the floor slab structure. On the other hand the soil under the slab will not dry out as efficiently as with the alternative laggings (B) and (C) because the temperature is lower. Alternative (A) is unfavourable with regard to the occurrence of ground frost in the foundations and its exclusive use is not to be recommended in soil that is susceptible to frost.

When the lagging is done according to alternative (B), the soil layers under the slab attain a relatively high temperature, which is favourable in view of their drying. Consequently, the thermal insulating capacity of the soil will improve. When this kind of lagging is applied, the 0 °C isothermal line cannot take its course through the layers under the house; it is therefore suitable also when the ground is susceptible to frost. When the ground water table is so close to the soil surface that the lagging sunk into the ground can become wet, alternative (B) is unfavourable.

Also alternative (C) prevents the access of frost to the soil layers under the house, but the lagging material is exposed to moisture and its insulating capacity will be impaired. This method can be used when the thermal insulation of existing houses has to be improved. On rock ground and wet soil this method gives poor results because these ground constituents have high thermal conductivity. In Finnish conditions the ground is covered by a well-insulating snow layer during the greater part of the heating period, and this in itself is equivalent to lagging according to alternative (C).

The lagging material should be water-repellent; it must not show capillary water absorption, nor should it deteriorate through the effects of moisture. It should also be able to carry the requisite load. Such materials are, for instance, light-weight gravel, foam glass and polystyrene. Also excelsior board, cinder stone and light-weight concrete have been successfully used.

322. Dimensioning of the thermal lagging

When a thermal lagging layer is provided under the floor slab, the soil surface temperature under the slab will not have a constant value T_o, as was assumed in deriving the equations for the isotherm field and the heat flow; the temperature will rather be lower towards the edges of the slab. The solution of the equations concerning the temperature distribution under an insulated slab and the heat flow from the slab into ground would require a different approach, using a set of boundary conditions specifying the surface temperature of the slab as a known function $T(x, y)$ instead of the constant value T_o. However, this would not essentially improve the accuracy of the results in actual practice, since the function $T(x, y)$, would have to be determined by approximate methods. It is also to be noted that

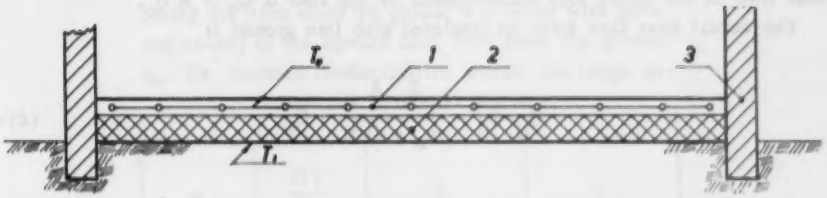

Fig. 50. Common method of lagging floor slabs. 1: Foundation slab, 2: Lagging course under the slab, 3: Edge lagging. T_0: Mean temperature at heating pipe level, T_1: Mean temperature under the lagging.

insulated slabs laid on loose soil constituents will be designed in principle as shown in Fig. 50, i.e., the lagging on the edges of the slab will be sunk to about 30 cm depth in the ground. The temperature T_1 under the lagging will then be approximately constant, which has also been corroborated by measurements, and the values of m_o in Table 4, p. 86, 5, p. 88 and 6, p. 89, which were calculated for constant surface temperature, can be used for the thermal resistance of the soil. These m_o values (or the k_o values in Table 7, p. 90) may be used after conversion to correspondence with the appropriate thermal conductivities, λ, of the soil.

The heat flow from the marginal zone into ground occurs within the lagging at the edges, which is sunk into the soil. It is therefore of small magnitude in itself. Its omission is further justified by the fact that the countersunk lagging causes a compensatory decrease of the heat flow into ground from the entire slab.

The total heat flow from an uninsulated slab into soil having the temperature 0 °C (without the field generated by the slab) at all points is

$$Q_{Go} = \frac{T_o\, A}{m_o} \tag{45}$$

and that from an insulated slab with a lagging having the thermal resistance Δm:

$$Q_G = \frac{T_o\, A}{m_o + \Delta m} \tag{46}$$

However, the soil temperature is not 0 °C, as the equations (45) and (46) would imply. The inherent temperature distribution field of the soil, $T_1(z)$, produces a thermal flow in the soil, which is directed upward during the cold season and has the magnitude q_o. According to the superposition principle, this quantity can be

deducted from the heat flow passing from the slab into the ground. The upward heat flow in the entire area encompassed by the slab is $Q_o = A\, q_o$.

The actual heat flow from an insulated slab into ground is

$$Q_G - Q_o = \frac{T_o\, A}{m_o + \Delta m} - Q_o \tag{47}$$

Considering equation (45), we find

$$Q_G - Q_o = \frac{m_o}{m_o + \Delta m}\, Q_{Go} - Q_o \tag{48}$$

This heat flow is less than the flow from the uninsulated slab into ground ($\Delta m > 0$); we can therefore write

$$Q_G - Q_o = \varkappa (Q_{Go} - Q_o), \text{ where } \varkappa < 1 \tag{49}$$

The coefficient $\varkappa$ characterizing the efficiency of the lagging is chosen in accordance with the required insulation; it indicates to which fraction the heat flow from the uninsulated slab is reduced by the lagging.

Equations (48) and (49) can be solved for the thermal resistance of the lagging course:

$$\Delta m = \left[\frac{1}{\varkappa + (1 - \varkappa)\, \dfrac{Q_o}{Q_{Go}}} - 1 \right] m_o \tag{50}$$

The apparent heat transmittance coefficient k_a of the insulated floor slab structure, which is equivalent to the normally used heat transmittance value in the equation

$$Q_G - Q_o = k_a\, A\, T_o$$

is obtained from equation (47):

$$k_a = \frac{Q_G - Q_o}{A T_o} = \frac{1}{m_o + \Delta m} - \frac{q_o}{T_o} \tag{51}$$

Table 8. Mean temperature gradients in the ground during the time when the ground is covered with snow, and density of the upward heat flow from the ground, q_o, for thermal conductivities within the range encountered in actual practice.

a	$\frac{\Delta T}{\Delta z}$	λ	q_o
m^2/h	$^oC/m$	kcal/m h ^oC	kcal/m^2 h
0,0022	2,86	0,5 ... 1,3	1,4 ... 3,7
0,0032	2,40	1,2 ... 2,0	2,9 ... 4,8
0,0047	1,96	3,0	5,9

In order to be able to determine the heat flow $q_o = -\lambda \frac{\Delta T}{\Delta z}$ arriving at the soil surface from the ground, one has to know the thermal conductivity of the soil and the temperature gradient in the soil. Table 8 gives the range of variation of this heat flow in snow-covered soil for three different values of the soil's thermal diffusivity (cf. Figs. 1, 2 and 3, pp. 15 ... 17).

The thermal resistance of a heat-insulated building, as calculated from soil surface to outdoor air, is approximately the same as that of a fairly thick snow layer. For instance, the thermal resistance of Experimental House No. 1 from the lower surface of the lagging under the slab to ambient air is m = 2,0 m^2 h ^oC/kcal, which is equivalent to a snow layer of about 33 cm thickness when the thermal conductivity of snow is λ = 0,06 kcal/m h ^oC. The lower surface of the lagging under the slab of an unheated house will therefore receive a heat flow equal in magnitude to that arriving at the surface of the surrounding ground. However, the warm slab causes the soil to dry out, reducing its thermal conductivity, in consequence of which also the heat flow density q_o in the region of the slab becomes less. The minimum values of the heat flow stated in Table 8 can be used as guiding values when the lagging of a slab is being dimensioned. The required efficiency of lagging will then be positively achieved.

The losses from the part of the floor slab structure above soil surface directly into ambient air have to be calculated separately (cf. p. 73).

Example No. 1

Experimental House No. 1, area of the floor slab: A = 103 m^2. Mean thermal conductivity of the soil: λ_G = 1,55 kcal/m h ^oC. Mean temperature of the slab: T_o = 29,5 ^oC. Permissible heat flow directly from the slab into ground shall be: Q_G = 840 kcal/h. – These values are consistent with the measurements in the said house during the time January 1st to March 31st, 1957 (Figs. 32, p. 57 and 45, p. 72).

We determine the required additional lagging Δm under the slab by equation (50), after which we compare it with the lagging that was actually applied.

For $A_o = 100\ m^2$, we find from Table 7: $k_o = 0{,}408$ kcal/m^2 h °C, as corresponding to $\lambda_o = 1$ kcal/m h °C. For the slab of size $A = 103\ m^2$, we find by equation (44):

$$k' = \frac{1}{\sqrt{\frac{103}{100}}} \cdot 0{,}408 \text{ kcal/m}^2\text{ h }^\circ\text{C} = 0{,}402 \text{ kcal/m}^2\text{h }^\circ\text{C}$$

Hence

$$m'_o = \frac{1}{k'} = \frac{1}{0{,}402} \frac{\text{m}^2\text{h }^\circ\text{C}}{\text{kcal}} = 2{,}488 \frac{\text{m}^2\text{h }^\circ\text{C}}{\text{kcal}}$$

We convert this by equation (43) so that we obtain the m_o value corresponding to the actual thermal conductivity of the soil, $\lambda = 1{,}55$ kcal/m h °C;

$$m_o = \frac{1}{1{,}55} \cdot 2{,}488 \frac{\text{m}^2\text{ h }^\circ\text{C}}{\text{kcal}} = 1{,}605 \frac{\text{m}^2\text{ h }^\circ\text{C}}{\text{kcal}}$$

The heat flow from the uninsulated slab into ground, according to equation (45), is

$$Q_{Go} = \frac{29{,}5 \cdot 103}{1{,}605} \text{ kcal/h} = 1893 \text{ kcal/h}$$

and the upward heat flow from the ground, according to Table 8:

$$Q_o = 2{,}9 \cdot 103 \text{ kcal/h} = 299 \text{ kcal/h}$$

from which the difference

$$Q_{Go} - Q_o = 1594 \text{ kcal/h}$$

The fractional proportion of the heat losses from the insulated and the uninsulated slab is

$$\varkappa = \frac{840}{1594} = 0{,}527$$

Equation (50) now gives the thermal resistance of the required additional lagging as

$$\Delta m = 1,06 \frac{m^2\,h\,^oC}{kcal}$$

The thermal conductivities of the lagging courses interposed between the slab of Experimental House No. 1 and the ground are known, and we can calculate their actual combined thermal resistance:

	d	λ	m
	m	kcal/m h °C	m^2 h °C/kcal
Concrete	0,26	1,0	0,26
Light-weight gravel	0,04	0,09	0,44
Shingle	0,10	0,35	0,29
			$\Delta m = 0,99$

The thermal resistance calculated by the dimensioning formulae is about 7 % higher than that computed for the actual structure. This is mainly due to the fact that q_o has been chosen from Table 8 slightly smaller than its real value for reasons of safety.

During the cold period, when the outdoor air temperature is fairly constant about -25 °C, the temperatures of outgoing and incoming heating water in Experimental House No. 1 are 50 °C and 44 °C, respectively, with the mean 47 °C. Fig. 41, p. 64 then gives T_o = 38,3 °C for the temperature at the heating pipe level. The heat flow from the slab under consideration into ground is then $Q_{Go} - Q_o$ = 2157 kcal/h when the slab is uninsulated and $Q_G - Q_o$ = 1222 kcal/h in the insulated condition, with Δm = 0,99 m^2 h °C/kcal.

Example No. 2

The apparent heat transmittance coefficient for Experimental House No. 1 is obtained as follows. – According to the preceding example, there is

$$m_o = 1,61 \frac{m^2\,h\,^oC}{kcal}$$
$$\Delta m = 0,99 \quad -"-$$
$$q_o = 2,90 \text{ kcal/m}^2\,h$$
$$T_o = 29,5\ ^oC$$

Insertion in equation (51) gives: k_a = 0,288 kcal/m^2 h °C, in good agreement with the value k_a = 0,284 kcal/m^2 h °C resulting from actual measurements (cf. p. 83).

Since the heat flow into ground is greater in the marginal area than in the central region of the slab, the area at the edges of the slab should preferably have a more efficient lagging. The thickness of the lagging course may also be calculated separately for various zones with the aid of the tables given in the foregoing, but the lagging layers on the edges of the slab will be somewhat overdimensioned by the calculating procedure that has been outlined above. This is due to the fact that the soil surface temperature under the lagging declines towards the edges. On the other hand it has to be noted that the soil has higher thermal conductivity on the edges of the slab than in the central regions. It is economically advantageous, particularly in large buildings and when the thermal conductivity of the soil is low, to place the lagging mainly under the marginal regions of the slab and under its corners, but the possibility of frost action has to be kept in mind in such cases.

4. SUMMARY

In two previous investigations concerning the heating of floors laid immediately on the ground, the author has dealt with the thermal conditions in undisturbed soil [16] and with the temperature distribution fields generated in the soil by warm slabs of different shapes which have been laid immediately on the ground, under different sets of boundary conditions [12].

The present work is a treatment, partially based on the said earlier publications, of the thermal conditions existing under warm floor slabs laid immediately on the ground and of the heat losses from the slab into ground, both theoretically and on the strength of results derived from measurements.

- The temperature distribution field $T(z, t)$ in snow-covered soil has been calculated by the same method which was applied to the case of bare soil in an earlier publication [16].
- The equations for the stationary temperature field produced in the soil by a circular slab, and for the heat flow, have been solved in connection with the present work under the boundary conditions best consistent with circumstances in actual practice. The solutions of the corresponding equations for the temperature fields generated in the soil by rectangular slabs and by slabs having the shape of a narrow strip, which are applied in practice in this work, have been presented in the author's earlier publication [12].
- In application of the theoretically derived results for the requirements encountered in practice, the typical temperature distribution fields existing under different kinds of floor slabs in wintertime conditions have been worked out by superimposing on the field produced by the slab the field consistent with the temperature distribution prevailing on the average in snow-covered soil during the period when houses are heated.
- Comparison of the non-stationary and stationary temperature distribution fields calculated for a slab having the shape of a narrow strip (i.e., for terrace houses)

by means of the equations derived by the authot in his previous work [12] revealed that the soil temperature field at greater depths does not attain a stationary condition during the heating period. However, the presented equations for the stationary temperature distribution field and heat flow can be used with adequate accuracy in practical applications. The stationary-state equations for the calculation of the heat flow are well applicable because the heat travels from the slab to the surrounding soil surface through the upper soil layers, in which approximately stationary conditions ensue within a relatively short period.

- Experimental investigations have been carried out in complementation of the theoretical results. Measurements have been performed in four experimental houses, three of them with a floor slab heated by tube elements embedded in the floor, while the floor of the fourth house derives its heat from the air of the room, which is heated by radiators. The results of measurement presented in this work include, among other things, the annual variation of soil temperatures in the region covered by the slab and in its neighbourhood at different depths, typical isotherm graphs for different seasons, and quantitative data concerning the heat flow from the slab to the ground. From the field graphs, inferences could be drawn with respect to the risk of ground frost action under buildings erected immediately on the ground. The moisture distribution in the soil under the slab in wintertime has also been presented. Finally, the thermal energy balance for the experimental house subjected to the most detailed studies has been compiled, showing the distribution of the heat quantities introduced into the house by the different heat loss groups and the contribution of the heat losses into ground to the total heat consumption of the house.

- Tables have been prepared, containing values calculated by the equations derived for computation of the heat losses from the foundation slabs of buildings into ground, applicable to slabs of different shapes and of given size. The values found in the tables can also be converted to apply to different combinations of slab size, temperatures and thermal conductivity of the soil by means of simple formulae given in the text.

- Equations have been derived for calculation of the heat losses from insulated slabs, enabling the precomputed tables to be used for calculating the heat losses into ground and for dimensioning thermal lagging of such slabs. Comparison of the values obtained by the theoretical equations and results of actual measurements revealed good agreement. The accuracy of the result of computation is primarily dependent on the accuracy with which the upward heat flow from the soil during the heating period, caused by the heat quantities stored in the ground during the summer, can be determined.

- In several connections secondary results relating to questions of the heating and moisture conditions of floors laid immediately on the ground have been presented. Such results are: the distribution of the floor surface temperature, the relationship between the mean temperature in the floor slab at the level of the heating tubes and the mean of the outgoing and incoming heating water temperatures, the

dependence of the relative humidity of the air in the pore spaces of the soil on the moisture content of the soil, the heating effect exerted by the sun, etc.

- Heating of the floor slab is a fairly pleasant method of heating from the viewpoint of dwelling comfort; a permanent resident of Experimental House No. 1 conrerned in this investigation, the author can attest to this from personal experience. This implies, however, sufficiently good insulation of the house so that floor surface temperatures of 27 ... 30 °C are not usually exceeded.

Considering the decisive influence on the heat losses from the floor slab to the ground and (in the case of non-stationary fields) also on the soil temperatures, exerted by the thermal conductivity of the soil, further research concerning this characteristic is required so that its values could be determined for the soil types occurring in Finland. It is thought to be desirable to carry out determinations of the thermal conductivity of the soil in addition to the soil examination, e. g. by the probe method [17], on building sites or in larger residential areas where buildings are to be erected immediately on the ground.

5. APPENDIX

For practical applications, universal nomographs permitting easy determination of the heat flow dissipated by slabs having various shape, dimensions and temperature into soils of various thermal conductivities can be devised from the equations representing the total thermal flow into ground from slabs having rectangular shape, the shape of a narrow strip, or circular shape.

For rectangular slabs, the dimensionless variable

$$\frac{Q}{\lambda T L} = f\left(\frac{b}{a'}, \frac{a' - a}{a}\right) \tag{1}$$

where $L = 2(a + b)$

is introduced in accordance with equation (5). This can be plotted as a function of $\frac{a' - a}{a}$, with b/a as parameter.

Similarly, a dimensionless variable

$$\frac{Q}{\lambda T L} = f\left(\frac{a' - a}{a}\right) \tag{2}$$

which is a function of $\frac{a' - a}{a}$ only, is used in accordance with equation (7) for slabs having the shape of a narrow strip. L is the length of the strip.

For circular slabs equation (24) suggests the choice of the dimensionless variable

$$\frac{Q}{\lambda T L} = f(\mu) \tag{3}$$

where $L = 2\pi R_o$. This is a function of $\mu = R_o/R$ only.

The calculations required in the drafting of these nomographs imply evaluation of the integrals in equations (5), (7) and (24) with the electronic computer for different values of the independent variable, and in equation (5) moreover at different parameter b/a values. In connection with the present work sufficient economic resources were not available for the use of an electronic computer on such large scale.

REFERENCES

1. Basements vs. no basements for houses, Washington 1948. [Housing and Home Finance Agency, Technical Bulletin No. 3 — 5]

2. Asplund, Sven Olof: Byggnader med uppvärmda golv på mark, Byggmästaren 32, (1953):2

3. Wredenfors, Walter: Det källarlösa husets teknik, Byggmästaren 34 (1955):10

4. Holmqvist, Nils: Värme- och fuktförhållanden vid golv- och grundkonstruktioner, VVS, Tidskrift för värme-, ventilations-, sanitets- och kylteknik 26 (1955):7

5. Ericsson, Hans: Hus utan källare - Grundläggningsmetoder, Norrköping 1958, [Statens nämnd för byggnadsforskning, Handlingar N:o 32]

6. Henriksson, Rolf: Värmeförluster genom golv lagda direkt på mark, Lund 1959, [Statens forskningsanstalt för lantmannabyggnader, Meddelande N:o 45]

7. Jansson, Ingvar, Holmqvist, Nils and Henriksson, Rolf: Isolering av golv och liggplatser i djurstallar, Lund 1957, [Statens forskningsanstalt för lantmannabyggnader, Meddelande N:o 38]

8. Humphreys, C. M., Franks, C. V and Schutrum, L. F: Field studies of heat losses from concrete floor panels, Heating piping & air conditioning, 23 (1951):1, pp. 181 — 186

9. Billington, N. S.: Electrical models for heating problems, J. Inst. heating ventilating engineers, Nov. 1950, pp. 247 — 261

10. Ruckli, Robert: Der Frost im Baugrund, Wien 1950, p. 217

11. Krischer, O.: Die Wärmeaufnahme der Grundflächen nicht unterkellerter Räume, Gesundheits-Ingenieur, 57 (1934):39, p. 513

12. Vuorelainen, O.: The temperature field produced in the ground by a heated slab laid direct on ground, and the heat flow from slab to ground, Helsinki 1960. [Acta Polytechnica Scandinavica. Civil Engineering and Building Construction Series No. 8]

13. Weyh, W.: Die Berechnung des Wärmeaustausches von Bodenflächen geheizter oder gekühlter Räume, Wärme- und Kältetechn. 38 (1936):11

14. Rietschel/Reiss: Heiz- und Lüftungstechnik, Wien 1958, pp. 368 – 369

15. Adlam, T. Napier: Radiant heating, New York 1940, Chapter 12

16. Vuorelainen, O.: Thermal conditions in the ground from the viewpoint of foundation work, heating and plumbing installations and draining, Helsinki 1960. [Acta Polytechnica Scandinavica. Civil Engineering and Building Construction Series No. 7]

17. Saare, Erik och Wenner, Carl-Gösta: Värmeledningstal hos olika jordarter, Norrköping 1957, [Statens nämnd för byggnadsforskning, Handlingar N:o 31]

18. Sneddon, Ian N.: Fourier transforms, New York, Toronto, London 1951, p. 48

19. Rydberg, Johan och Arnell, Åke: Ventilationens storlek i bostäder (The rate of ventilation in dwellings), Stockholm 1949, [Statens nämnd för byggnadsforkning, Meddelande N:o 15]

20. Chrenko, F. A.: The effects of the temperatures of the floor surface and of the air on thermal sensations and the skin temperature of the feet, Brit. J. industr. med. 14 (1957):13

21. Newis, R.G., Flinner, A.O.: Effects of heated floor temperatures on comfort, Heating, piping & air conditioning 29 (1957):10, p. 149

22. Newburgh, L.H.: Physiology of heat regulation and the science of clothing, Philadelphia, London 1949, pp. 448 and 194

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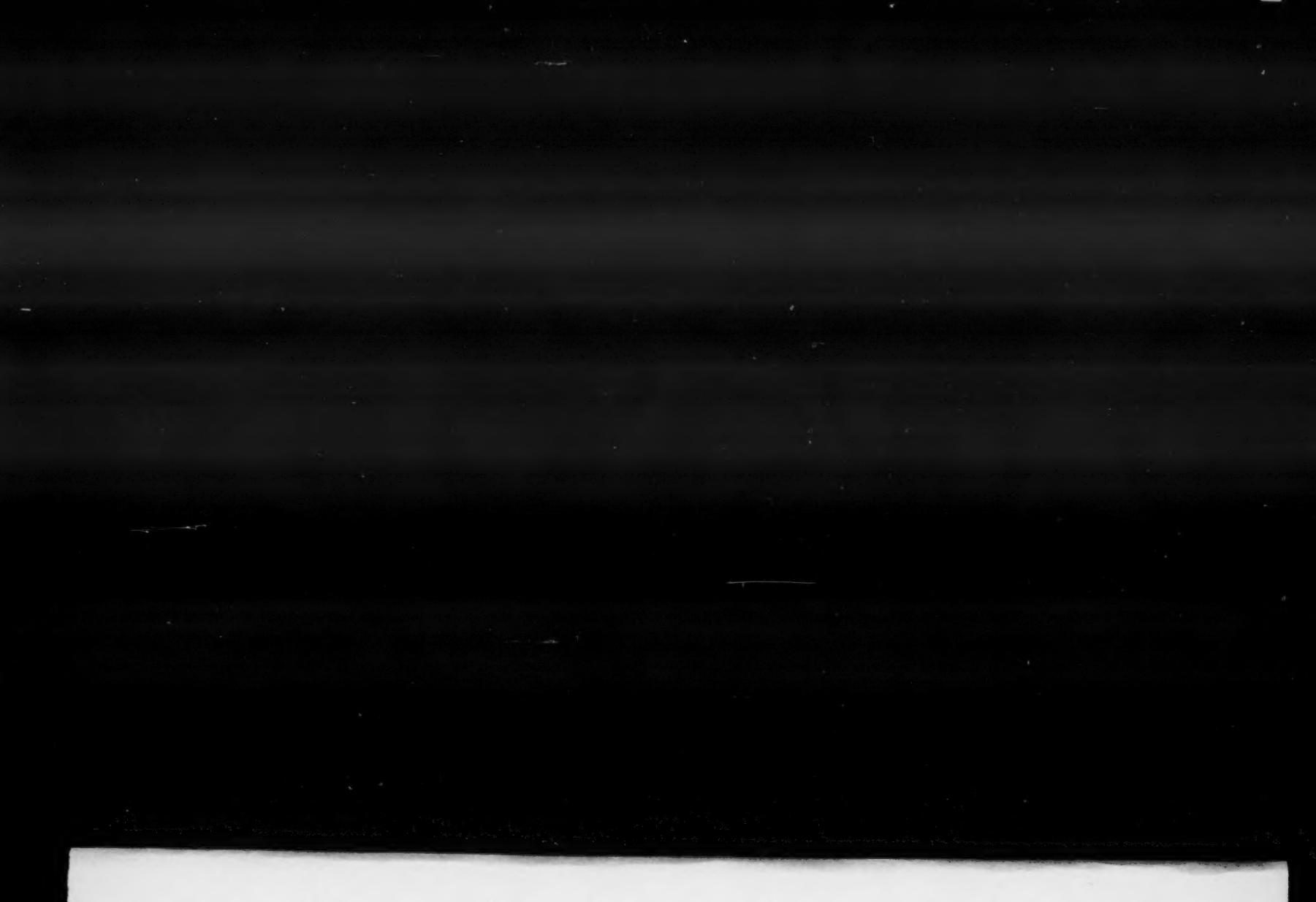
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